

Tutorial 2

$$1. \quad f(t) = \frac{1}{4+t^2} \quad F(\omega) = \frac{\pi}{2} e^{-2|\omega|}$$

$$(a) \quad f(t-4) \rightarrow e^{-4i\omega} F(\omega) = \frac{\pi}{2} e^{-4i\omega - 2|\omega|}$$

$$(b) \quad f(3t) \rightarrow \frac{1}{3} F\left(\frac{\omega}{3}\right) = \frac{\pi}{6} e^{-\frac{2}{3}|\omega|}$$

$$(c) \quad f(3t-12) = f(3(t-4)) \\ \rightarrow e^{-4i\omega} \text{FT}\{f(3t)\} = e^{-4i\omega} \frac{1}{3} F\left(\frac{\omega}{3}\right) = e^{-4i\omega} \frac{\pi}{6} e^{-\frac{2}{3}|\omega|}$$

$$(d) \quad f(2t+1) = f(2(t+\frac{1}{2})) \\ \rightarrow e^{i\omega/2} \text{FT}\{f(2t)\} = e^{i\omega/2} \frac{1}{2} F\left(\frac{\omega}{2}\right) = e^{i\omega/2} \frac{\pi}{4} e^{-|\omega|}$$

$$(e) \quad e^{3it} f(t) \rightarrow F(\omega-3) = \frac{\pi}{2} e^{-2|\omega-3|}$$

$$(f) \quad \frac{df}{dt} \rightarrow i\omega F(\omega) = \frac{i\omega\pi}{2} e^{-2|\omega|}$$

$$(g) \quad F(\omega) \rightarrow 2\pi f(-\omega) = 2\pi \frac{1}{4+(-\omega)^2} = \frac{2\pi}{4+\omega^2}$$

$$(h) \quad t f(t) \rightarrow i \frac{dF}{d\omega} = \frac{i\pi}{2} \frac{d}{d\omega} e^{-2|\omega|} \\ = \frac{i\pi}{2} \cdot -2 \operatorname{sgn}(\omega) e^{-2|\omega|} \\ = -i\pi \operatorname{sgn}(\omega) e^{-2|\omega|}$$

$$2 \quad f(t) = e^{-t} u(t) \quad F(\omega) = \frac{1}{1+i\omega}$$

$$\frac{1}{1+it} \equiv F(t)$$

$$\text{FT} \{F(t)\} = 2\pi f(-\omega) = 2\pi e^{\omega} u(-\omega)$$

$$\frac{1}{1-it} = -\frac{1}{it-1}$$

$$f_1(t) = -e^t u(-t)$$

$$F_1(\omega) = \frac{1}{i\omega-1}$$

$$F_1(t) = \frac{1}{it-1}$$

$$\begin{aligned} \text{FT} \{F_1(t)\} &= 2\pi f_1(-\omega) \\ &= -2\pi e^{-\omega} u(\omega) \end{aligned}$$

$$\therefore \text{FT} \left\{ \frac{1}{1-it} \right\} = 2\pi e^{-\omega} u(\omega)$$

$$\begin{aligned} \frac{1}{1+it} + \frac{1}{1-it} &\rightarrow 2\pi e^{\omega} u(-\omega) + 2\pi e^{-\omega} u(\omega) \\ &= 2\pi e^{-|\omega|} \end{aligned}$$

$$3 \text{ (ii) let } F(\omega) = \frac{1}{1+i\omega} \Rightarrow f(t) = e^{-t} u(t)$$

$$\frac{1}{1+(i\omega+\gamma)i} = F(\omega+\gamma)$$

$$\text{FT}^{-1} [F(\omega+\gamma)] = e^{-\gamma t} f(t)$$

$$= e^{-\gamma t - t} u(t)$$

$$(ii) \quad \frac{2}{1 + 2(\omega - 1)i} = \frac{1}{\frac{1}{2} + (\omega - 1)i}$$

$$\text{Let } F(\omega) = \frac{1}{\frac{1}{2} + \omega i} \Rightarrow f(t) = e^{-\frac{1}{2}t} u(t)$$

$$\frac{1}{\frac{1}{2} + (\omega - 1)i} = F(\omega - 1)$$

$$\therefore \mathcal{FT}^{-1}[F(\omega - 1)] = e^{it} f(t) = e^{it - \frac{1}{2}t} u(t)$$

$$\begin{aligned} 4 (i) \quad F(\omega) &= 6 e^{-4i\omega} \frac{\text{sinc}(\omega)}{\omega} = 6 e^{-4i\omega} \text{sinc}(\omega) \\ &= 6 e^{-4i\omega} \text{sinc}\left(\frac{\omega \cdot 2}{2}\right) \\ &= 3 \cdot e^{-4i\omega} \cdot 2 \text{sinc}\left(\frac{2\omega}{2}\right) \end{aligned}$$

$$\text{Let } G(\omega) = 2 \text{sinc}\left(\frac{2\omega}{2}\right) \Rightarrow g(t) = g_2(t) = u(t+1) - u(t-1)$$

$$f(t) = \mathcal{FT}^{-1} \left\{ 3 e^{-4i\omega} G(\omega) \right\}$$

$$= 3 g(t-4) = 3 g_2(t-4) = 3 [u(t-3) - u(t-5)]$$

$$(iii) \quad G(\omega) = e^{-2\omega^2} = \frac{1}{\sqrt{8\pi}} \frac{\sqrt{\pi}}{\sqrt{1/8}} e^{-\frac{\omega^2}{4(1/8)}}$$

$$\therefore g(t) = \frac{1}{\sqrt{8\pi}} e^{-\frac{1}{8}t^2}$$