

Tutorial 3

$$(a) \quad \sin(3t) \rightarrow \frac{\pi}{i} (\delta(\omega-3) - \delta(\omega+3))$$

$$(b) \quad t \sin(3t) \rightarrow j \frac{d}{d\omega} \frac{\pi}{i} (\delta(\omega-3) - \delta(\omega+3)) \\ = \pi (\delta'(\omega-3) - \delta'(\omega+3))$$

$$(c) \quad \delta'(t) \rightarrow i\omega$$

$$(d) \quad u(t-3) + t^2 \\ u(t-3) \rightarrow e^{-3i\omega} \text{FT}\{u(t)\} = e^{-3i\omega} \left[\pi\delta(\omega) + \frac{1}{i\omega} \right] \\ t^2 \rightarrow 2\pi i^2 \delta^{(2)}(\omega) = -2\pi \delta^{(2)}(\omega) \\ \therefore u(t-3) + t^2 \rightarrow e^{-3i\omega} \left[\pi\delta(\omega) + \frac{1}{i\omega} \right] - 2\pi \delta^{(2)}(\omega)$$

$$(e) \quad \sin(3t+4) = \sin(3(t+4/3)) \\ \rightarrow e^{4i\omega/3} \text{FT}\{\sin(3t)\} \\ = e^{4i\omega/3} \frac{\pi}{i} [\delta(\omega-3) - \delta(\omega+3)]$$

$$(f) \quad f(t) = u(t) \Rightarrow F(\omega) = \pi\delta(\omega) + \frac{1}{i\omega}$$

$$F(t) = \pi\delta(t) + \frac{1}{it} \quad \text{FT}\{F(t)\} = 2\pi f(-\omega) \\ = 2\pi u(-\omega)$$

$$(g) \quad e^{it} u(t+1) \rightarrow F(\omega-1) \text{ where } F(\omega) = \text{FT}\{u(t+1)\} \\ \text{FT}\{u(t+1)\} = e^{i\omega} \text{FT}\{u(t)\} = e^{i\omega} \left[\pi\delta(\omega) + \frac{1}{i\omega} \right] \\ \therefore e^{it} u(t+1) \rightarrow e^{i(\omega-1)} \left[\pi\delta(\omega-1) + \frac{1}{i(\omega-1)} \right]$$

$$(h) \quad e^{-4it} \cos(5t) \rightarrow F(\omega+4) \\ \text{where } F(\omega) = \text{FT}\{\cos(5t)\} = \pi [\delta(\omega-5) + \delta(\omega+5)] \\ \therefore e^{-4it} \cos(5t) \rightarrow \pi [\delta(\omega-1) + \delta(\omega+9)]$$

$$2. \quad \sin(3\omega + 4) = \sin(3(\omega + 4/3))$$

$$= F(\omega + 4/3) \quad \text{say}$$

$$\rightarrow e^{-4i/3t} f(t)$$

$$\text{where } f(t) = FT^{-1}[\sin(3\omega)]$$

$$\text{if } g(t) = \sin(3t) \Rightarrow G(\omega) = \frac{\pi}{i} [\delta(\omega - 3) - \delta(\omega + 3)]$$

$$G(t) = \frac{\pi}{i} [\delta(t - 3) - \delta(t + 3)]$$

$$FT \{G(t)\} = 2\pi g(-\omega) = 2\pi \sin(-3\omega)$$

$$= -2\pi \sin(3\omega)$$

$$\therefore FT^{-1} \{ \sin(3\omega) \} = \frac{-1}{2\pi} G(t) = -\frac{1}{2i} [\delta(t - 3) - \delta(t + 3)]$$

$$\therefore \sin(3\omega + 4) \rightarrow -\frac{e^{-4it/3}}{2i} [\delta(t - 3) - \delta(t + 3)]$$

$$(h) \quad \delta'(\omega - 3) = F(\omega - 3) \quad \text{where } f(\omega) = \delta'(\omega)$$

$$f(t) = FT^{-1}[\delta'(\omega)] = \frac{t}{2\pi i}$$

$$F(\omega - 3) \rightarrow e^{3it} f(t)$$

$$\therefore \delta'(\omega - 3) \rightarrow e^{3it} \cdot \frac{t}{2\pi i}$$

$$(c) \quad u(\omega + 4) = F(\omega + 4) \quad \text{where } F(\omega) = u(\omega)$$

$$g(t) = u(t) \Rightarrow G(\omega) = \pi \delta(\omega) + \frac{1}{i\omega}$$

$$G(t) \Rightarrow 2\pi g(-\omega) = 2\pi u(-\omega)$$

$$FT^{-1}[u(-\omega)] = \frac{1}{2\pi} \left[\pi \delta(t) + \frac{1}{it} \right]$$

$$FT^{-1}[u(\omega)] = \frac{1}{2\pi} \left[\pi \delta(-t) - \frac{1}{it} \right] = \frac{1}{2\pi} \left[\pi \delta(t) - \frac{1}{it} \right]$$

$$u(\omega + 4) \rightarrow e^{-4it} \frac{1}{2\pi} \left[\pi \delta(t) - \frac{1}{it} \right]$$