

①

$$\begin{array}{l} S_1 \quad 0.625 \\ S_2 \quad 0.25 \\ S_3 \quad 0.125 \end{array} \left. \vphantom{\begin{array}{l} S_1 \\ S_2 \\ S_3 \end{array}} \right\} 0.625$$

Reversing

$$\begin{array}{l} S_1 \quad 0.625 \\ S_2 \quad 0.25 \\ S_3 \quad 0.125 \end{array} \left. \vphantom{\begin{array}{l} S_1 \\ S_2 \\ S_3 \end{array}} \right\} \begin{array}{l} 0 \\ 00 \\ 11 \end{array} \quad 0.625$$

$$L = 0.625 + 2(0.25) + 2(0.125) = 1.375$$

$$H(s) = - \left\{ 0.625 \frac{\ln(0.625)}{\ln(2)} + 0.25 \log_2(0.25) + 0.125 \log_2(0.125) \right\}$$

$$= 0.423794 + 0.5 + 0.375$$

$$= \underline{1.29879}$$

(2)

 $P_2(i)$

$$\begin{array}{lcl}
S_1 S_1 & 0.390625 & \xrightarrow{0} 0.390625 \\
S_1 S_2 & 0.15625 & \xrightarrow{100} 0.15625 \\
S_2 S_1 & 0.15625 & \xrightarrow{101} 0.15625 \\
S_1 S_3 & 0.078125 & \xrightarrow{1100} 0.078125 \\
S_3 S_1 & 0.078125 & \xrightarrow{1101} 0.078125 \\
S_2 S_2 & 0.0625 & \xrightarrow{1110} 0.0625 \\
S_2 S_3 & 0.03125 & \xrightarrow{11110} 0.03125 \\
S_3 S_2 & 0.03125 & \xrightarrow{111110} 0.046875 \\
S_3 S_3 & 0.015625 & \xrightarrow{111111} 0.015625
\end{array}$$

$$\begin{array}{lcl}
0 & 0.390625 & \\
100 & 0.3125 & \\
101 & 0.296875 & \\
11 & 0.296875 &
\end{array}$$

$$L_2 = \frac{1}{2} \leq P_i L_i$$

$$= \frac{1}{2} (2.640) = 1.3203$$

$$\text{cf. } L_1 = 1.375 \text{ and}$$

$$H(S) \approx 1.29879$$

$$H(2S) \approx 2.597589 \quad H(2S) = 2.145,$$

③ 2nd Extension

$$\begin{array}{rcl}
 S_1 S_1 & 0.16 & \begin{array}{l} 00 \\ 01 \end{array} \rightarrow 0.4 \\
 S_1 S_2 & 0.24 & \begin{array}{l} 10 \\ 11 \end{array} \rightarrow 0.6 \\
 S_2 S_1 & 0.24 & \begin{array}{l} 00 \\ 01 \end{array} \rightarrow 0.4 \\
 S_2 S_2 & 0.36 & \begin{array}{l} 10 \\ 11 \end{array} \rightarrow 0.6
 \end{array}$$

$$L_2 = 2(0.16 + 0.24 + 0.24 + 0.36) = 1 \text{ unchanged.}$$

original Code

$$\begin{array}{l}
 S_1: 0.4 \\
 S_2: 0.6
 \end{array}
 \quad L = 1 \times 0.4 + 1 \times 0.6 = 1$$

$$\begin{array}{rcl}
 S_1 S_1 S_1 & 0.064 & \begin{array}{l} 000 \\ 001 \end{array} \rightarrow 0.16 \\
 S_1 S_1 S_2 & 0.096 & \begin{array}{l} 100 \\ 101 \end{array} \rightarrow 0.24 \\
 S_1 S_2 S_1 & 0.096 & \begin{array}{l} 010 \\ 011 \end{array} \rightarrow 0.24 \\
 S_2 S_1 S_1 & 0.096 & \begin{array}{l} 000 \\ 001 \end{array} \rightarrow 0.16 \\
 S_2 S_2 S_1 & 0.144 & \begin{array}{l} 100 \\ 101 \end{array} \rightarrow 0.288 \\
 S_2 S_1 S_2 & 0.144 & \begin{array}{l} 010 \\ 011 \end{array} \rightarrow 0.288 \\
 S_1 S_2 S_2 & 0.144 & \begin{array}{l} 000 \\ 001 \end{array} \rightarrow 0.216 \\
 S_2 S_2 S_2 & 0.216 & \begin{array}{l} 100 \\ 101 \end{array} \rightarrow 0.432
 \end{array}$$

$$\begin{aligned}
 L_3 &= 4(0.064 + 0.096) + 3(0.096 + 0.096 + 0.144 + 0.144 + 0.144) + 2(0.216) \\
 &= 0.9813
 \end{aligned}$$

$$\text{(Note } H(S) = -0.4 \log_2(0.4) - 0.6 \log_2(0.6) = 0.97095 \text{)}$$

4.

$$\text{Let } f(x) = \ln(x) - x + 1$$

$$f'(x) = \frac{1}{x} - 1 = 0 \quad \text{when } x = 1$$

$$f''(x) = -\frac{1}{x^2} < 0 \quad \text{so we have a maximum at } x = 1.$$

$$f(1) = \ln(1) - 1 + 1 = 0 - 1 + 1 = 0$$

$$\text{Thus } f(x) \leq 0 \Rightarrow \underline{\ln(x) \leq x - 1}$$