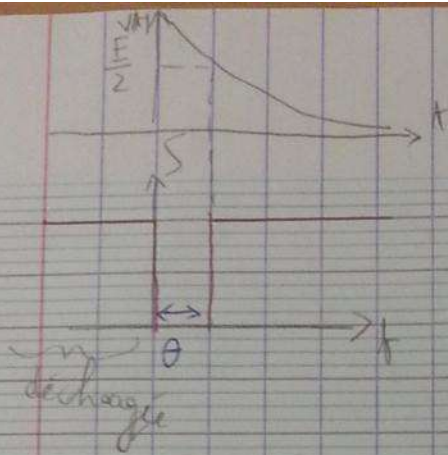




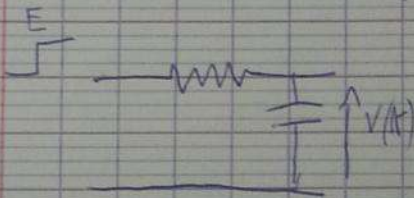
$$V(t) = E e^{-t/\tau} = \frac{E}{2}$$

$$t/\tau = 1$$

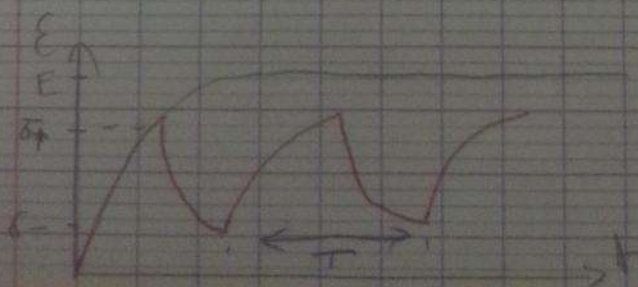
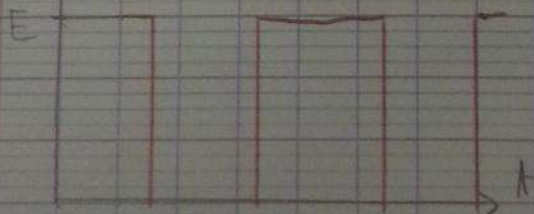
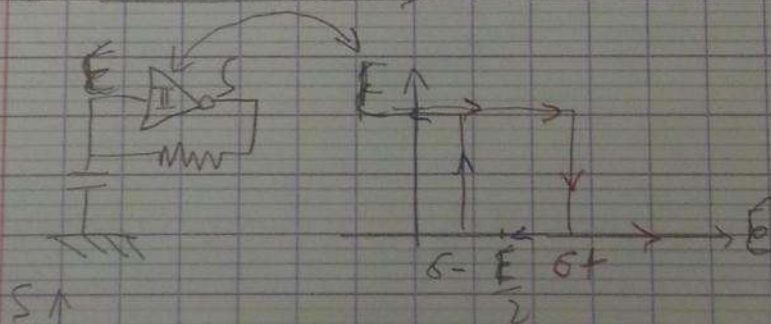
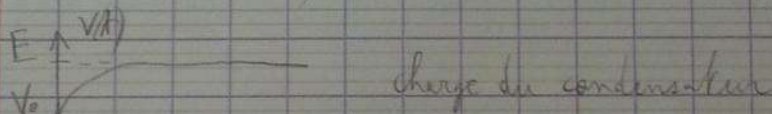
$$\tau = RC \ln 2$$

Ex basique astable

- * 1 circuit ML à hysteresis
- * charge/décharge

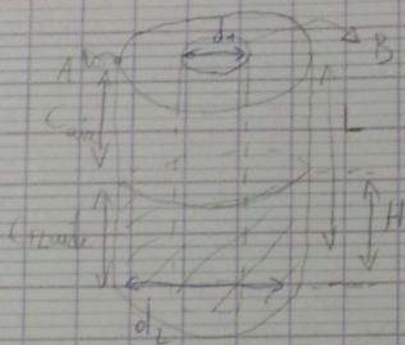


$$V(t) = E(1 - e^{-t/\tau}) + V_0 e^{-t/\tau}$$



$$T = 2RC \ln \left(\frac{E - \sigma_-}{E - \sigma_+} \right)$$

TD3 : II mesure de niveau

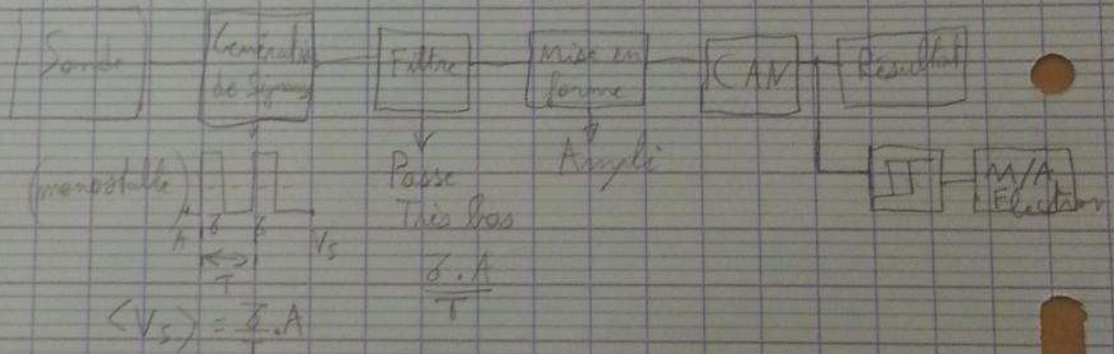


$$C = \frac{2\pi\epsilon_0\epsilon_r \cdot L}{\ln\left(\frac{d_2}{d_1}\right)}$$

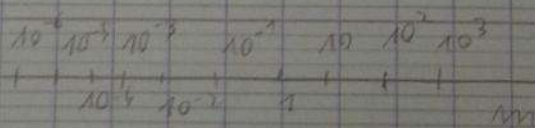
$$\begin{aligned} C_{AB} &= C_{AIR} // C_{FLUID} \\ &= C_{AIR} + C_{FLUID} \\ &= \frac{2\pi\epsilon_0}{\ln\left(\frac{d_2}{d_1}\right)} \left(\epsilon_{AIR} (L-H) + \epsilon_{FLUID} H \right) \end{aligned}$$

$$= 6,28 \times 8,85 \cdot 10^{-12} (1 + H)$$

$$= 50(H+1) pF$$



Capteurs Inductifs (mesures cinématiques)

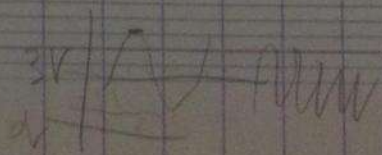


Capac < $\pm 90^\circ$

Profil < $\pm 350^\circ$

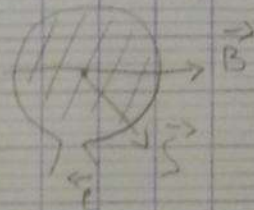
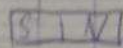
Induct < $\pm 30 \text{ à } 60^\circ$

Capacitif
résistance de
Fourniture



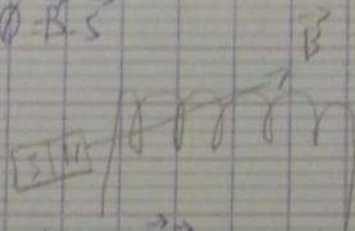
A) Principe d'induction, auto-induction, Mutuelle induction

* Induction



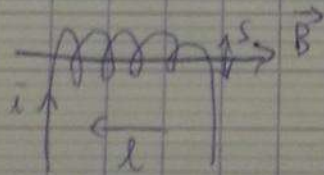
$$\epsilon = \frac{d\Phi}{dt}$$

$$\Phi = B \cdot S$$



$$\Phi = n \cdot B \cdot S$$

* Auto-Induction



$$\Phi = L i$$

$$L = \mu \frac{S n^2}{l}$$

(Henry)

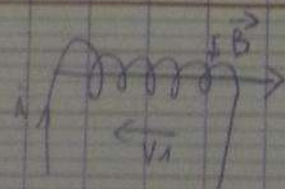
$$i \Rightarrow B \Rightarrow \Phi \Rightarrow l$$

$$\epsilon = \frac{d\Phi}{dt} = L \cdot \frac{di}{dt}$$

S: section
l: longueur

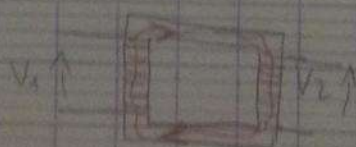
n = spires
μ = μ₀ μᵣ
μ₀ = 4π × 10⁻⁷ N/A²

* Mutuelle induction



(gpc du transfert)

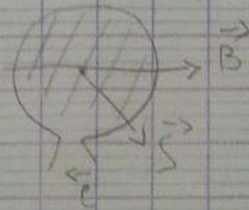
$$V_2 = M \frac{di_1}{dt}$$



A) Principe d'induction, auto-induction, Mutuelle induction

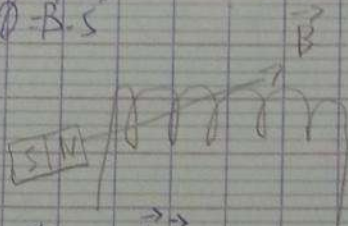
* Induction

S | N



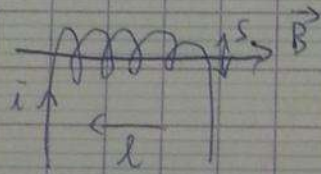
$$e = - \frac{d\Phi}{dt}$$

$$\Phi = \vec{B} \cdot \vec{S}$$



$$\Phi = n \cdot \vec{B} \cdot \vec{S}$$

* Auto-Induction



$$\Phi = L i$$

$$L = \mu \frac{S n^2}{l} \quad (\text{Henry})$$

$$i \Rightarrow B \Rightarrow \Phi \Rightarrow l$$

$$e = - \frac{d\Phi}{dt} = - L \frac{di}{dt}$$

S: section

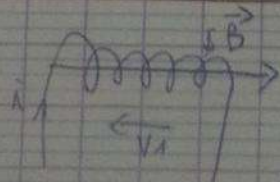
l: longueur

n = spires

$\mu = \mu_0 \mu_r$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$$

* Mutuelle induction

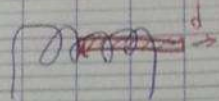


(gpe des transformateurs)

$$V_2 = M \frac{di_1}{dt}$$



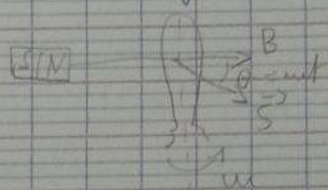
capteur à variation de L



μ varie
 $\rightarrow L$ varie avec d

capteur à variation de φ

Ex: Tachymètre.



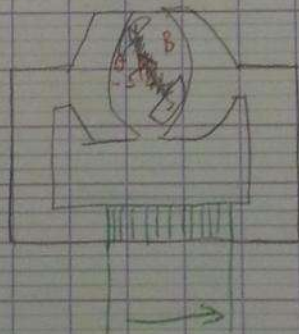
$$\varphi = BS \cos(\omega t)$$

$$e = -BS \omega \sin(\omega t)$$

Ex: μ Phase Electrodynamique

2) Exemple de réalisation.

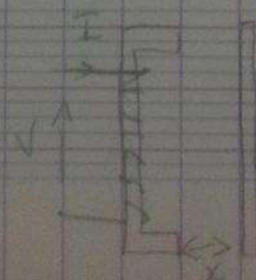
* Tacho



$$V_s = BS, \Omega(t), \sin \theta$$

$$\Omega(t) = \frac{d\theta}{dt}$$

3) Mesure petit déplacements



Annuaire

Terre - Magnétique mobile

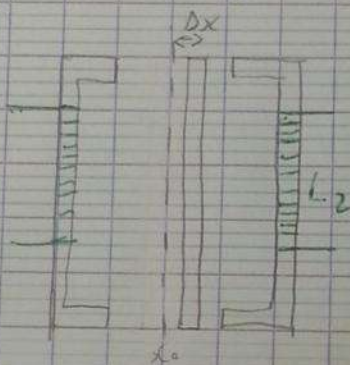
$$V = \frac{1}{2} \frac{dL}{dx} \quad L(x) = \frac{1}{2} N^2 \mu_0 S \frac{1}{x}$$

$$x = x_0 + \Delta x \quad x_0: \text{Ref}$$

$$L(x) = L_0 \cdot \frac{1}{1 + \frac{\Delta x}{x_0}}$$

$$L_0 = L(x_0) = \frac{1}{2} N^2 \mu_0 S \frac{1}{x_0}$$

$$L(x) = L_0 \left(1 - \frac{\Delta x}{x_0} + \left(\frac{\Delta x}{x_0} \right)^2 - \left(\frac{\Delta x}{x_0} \right)^3 \right)$$



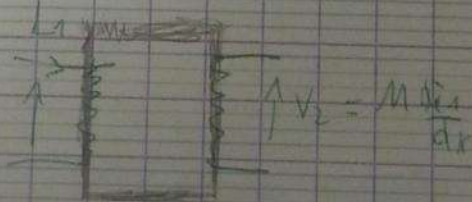
$$L_1(x) = L_0 \left(1 - \frac{\Delta x}{x_0} + \left(\frac{\Delta x}{x_0} \right)^2 - \left(\frac{\Delta x}{x_0} \right)^3 \right)$$

$$L_2(x) = L_0 \left(1 + \frac{\Delta x}{x_0} + \left(\frac{\Delta x}{x_0} \right)^2 + \left(\frac{\Delta x}{x_0} \right)^3 \right)$$

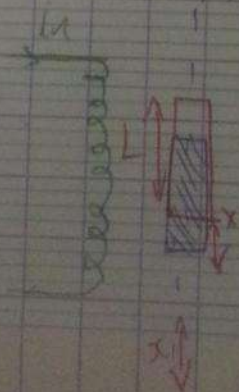
$$L_2(x) - L_1(x) = 2 L_0 \left(\frac{\Delta x}{x_0} + \left(\frac{\Delta x}{x_0} \right)^3 \right)$$

Améliore linéarité et $\uparrow$ sensible

4) Mesure de dpt avec LVDST (Linéaire Voltage Differential Transformer)
(Pro de variation de Mutuelle)



LVDST



$$\uparrow e_1 = M(L-x) \frac{di_1}{dt}$$

$$\downarrow e_2 = M(L+x) \frac{di_1}{dt}$$

Conditionnement :

$$\frac{e_1 - e_2}{e_1 + e_2} = \frac{M(L-x) - M(L+x)}{M(L-x) + M(L+x)} \quad \text{indépendant de } x$$

Tant que 1 DL pour $x \rightarrow 0$

$$f(x) = f(0) + f'(0) \cdot x \quad f(0) = 0$$

$$f(x) = f'(0) \cdot x$$

$$f'(0) = ?$$

$$f(x) = \frac{M(L-x) - M(L+x)}{M(L-x) + M(L+x)}$$

$$f'(x) = \frac{[-M'(L-x) - M'(L+x)][(M(L-x) + M(L+x))] - [-M'(L-x) + M'(L+x)][M(L-x) - M(L+x)]}{[M(L-x) + M(L+x)]^2}$$

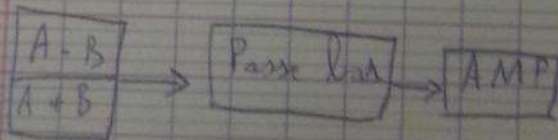
$$f'(x) = \frac{-2M'(L) \cdot 2M(L)}{(2M(L))^2} \Rightarrow f(x) = -\frac{M'(L)}{M(L)} x + \dots$$

$$f(x) = -\frac{M'(L)}{M(L)} x + \dots$$

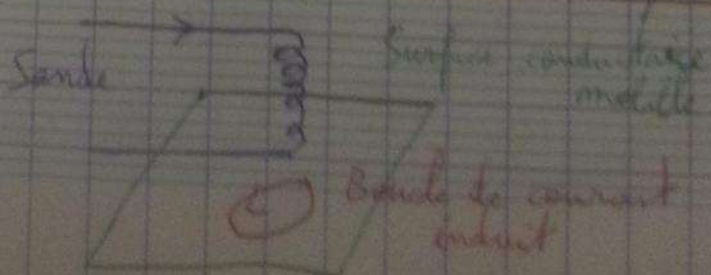
coef const. α

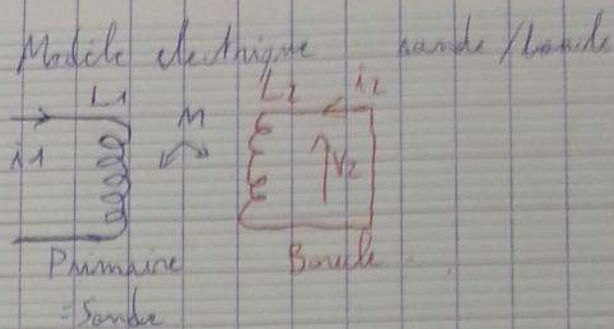
Non linéaire

GHF



5) Capteur à courant de Foucault : capteur sans contact





$$\begin{cases} v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M(x) \frac{di_2}{dt} \\ 0 = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{cases}$$

Régime sinusoïdal $\frac{d}{dt} \Leftrightarrow j\omega$

$$\begin{cases} v_1 = (R_1 + jL_1 \omega) I_1 + jM \omega I_2 \\ 0 = (R_2 + jL_2 \omega) I_2 + jM \omega I_1 \end{cases} \Rightarrow I_2 = \frac{-jM \omega I_1}{R_2 + jL_2 \omega}$$

$$\Rightarrow v_1 = \left[(R_1 + jL_1 \omega) + \frac{(M \omega)^2}{R_2 + jL_2 \omega} \right] I_1$$

Impédance de la source

$$\Rightarrow Z_c = R_1 + jL_1 \omega + \frac{(M \omega)^2 (R_2 - jL_2 \omega)}{R_2^2 + (L_2 \omega)^2}$$

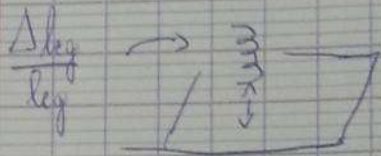
$$\Rightarrow Z_c = \left(R_1 + \frac{R_2 (M \omega)^2}{R_2^2 + (L_2 \omega)^2} \right) + j\omega \left(L_1 - \frac{L_2 (M \omega)^2}{R_2^2 + (L_2 \omega)^2} \right)$$

Résistance équivalente
Rc

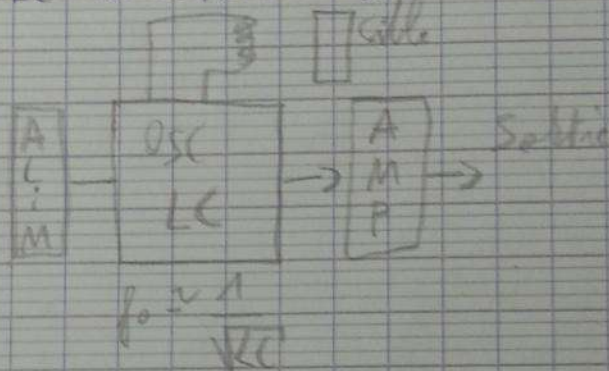
inductance équivalente
Lc

Pb à considérer au moment de la mise en œuvre $x \nearrow M, y \nearrow L_c \nearrow$

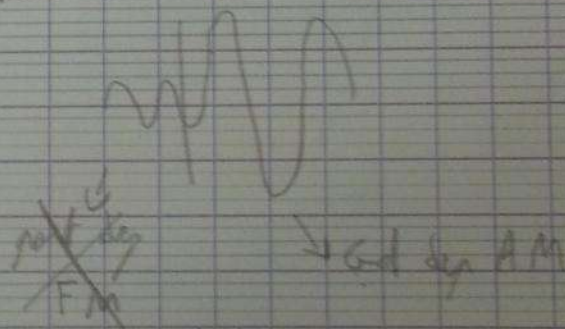
Conditionnement des CCF



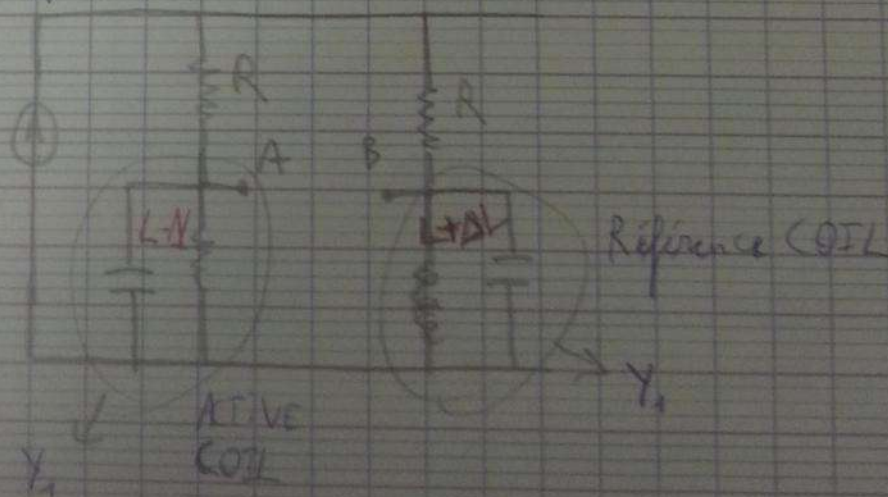
1) Le KD 2440 : "Base" : Osc "LC"



Tous N.L.



3) Le pont de capteurs différentiels KD 5100
(cf. TD4, Q4)



Capteurs à courant de Foucault

Pour simplifier, oublions les axes

$$L = L_0 + \Delta L = L_0 \left(1 + \frac{\Delta L}{L_0} \right)$$

$$Y = j\omega m + \frac{1}{jL\omega} = j\omega m + \frac{1}{jL_0 \left(1 + \frac{\Delta L}{L_0} \right) \omega}$$

$$Y = j\omega m + \frac{1}{jL_0\omega} \left(1 - \frac{\Delta L}{L_0} \right)$$

$$Y_2 = j\omega m + \frac{1}{jL_0\omega} \left(1 - \frac{\Delta L}{L_0} \right)$$

$$Y_1 = j\omega m + \frac{1}{jL_0\omega} \left(1 + \frac{\Delta L}{L_0} \right)$$

Sortie du pont

$$V_{AB} = \left(\frac{Z_1}{R+Z_1} - \frac{Z_2}{R+Z_2} \right) E$$

$$= \left(\frac{1}{1+RX_1} - \frac{1}{1+RX_2} \right) E$$

$$= \frac{R(Y_2 - Y_1)E}{1+R(Y_1+Y_2) + R^2 Y_1 Y_2}$$

$$\text{Or } Y_2 - Y_1 = -\frac{2\Delta L}{L_0}$$

$$Y_1 + Y_2 = 2 \left(j\omega m + \frac{1}{jL_0\omega} \right)$$

$$Y_1 Y_2 = \left(j\omega m + \frac{1}{jL_0\omega} \right)^2 - \left(\frac{\Delta L}{L_0} \right)^2$$

$$L = L_0 + \Delta L = L_0 \left(1 + \frac{\Delta L}{L_0} \right)$$

$$Y = g_{cm} + \frac{1}{g L_{cm}} = g_{cm} + \frac{1}{g L_0 \left(1 + \frac{\Delta L}{L_0} \right) m}$$

$$Y \approx g_{cm} + \frac{1}{g L_{cm}} \left(1 - \frac{\Delta L}{L_0} \right)$$

$$\begin{cases} Y_2 = g_{cm} + \frac{1}{g L_{cm}} \left(1 - \frac{\Delta L}{L_0} \right) \\ Y_1 = g_{cm} + \frac{1}{g L_{cm}} \left(1 + \frac{\Delta L}{L_0} \right) \end{cases}$$

Sortie du pont :

$$V_{AB} = \left(\frac{Z_1}{R+Z_1} - \frac{Z_2}{R+Z_2} \right) E$$

$$= \left(\frac{1}{1+RY_1} - \frac{1}{1+RY_2} \right) E$$

$$= \frac{R(Y_2 - Y_1) E}{1 + R(Y_1 + Y_2) + R^2 Y_1 Y_2}$$

$$\text{Or } Y_2 - Y_1 = -\frac{2 \Delta L}{L_0}$$

$$Y_2 + Y_1 = 2 \left(g_{cm} + \frac{1}{g L_{cm}} \right)$$

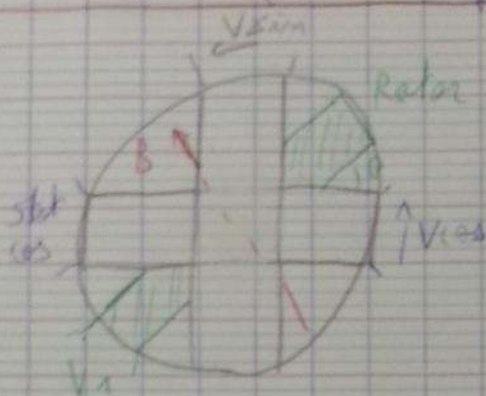
$$Y_1 Y_2 = \left(g_{cm} + \frac{1}{g L_{cm}} \right)^2 - \left(\frac{\Delta L}{L_0} \right)^2$$

$$\Rightarrow V_{AB} = \frac{-2R \frac{\Delta L}{L_0} \cdot E}{1 + 2 \left(g_{cm} + \frac{1}{g L_{cm}} \right) + \left(g_{cm} + \frac{1}{g L_{cm}} \right)^2}$$

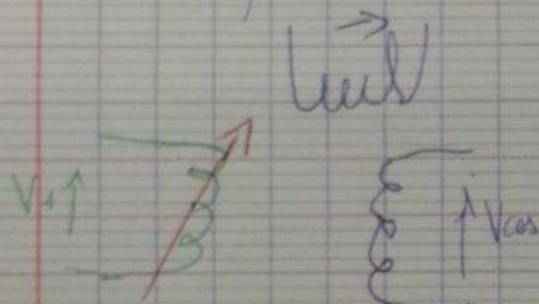
$$\Rightarrow V_{AB} = \frac{-2R \left(\frac{\Delta L}{L_0} \right) \cdot E}{\left[1 + \left(g_{cm} + \frac{1}{g L_{cm}} \right) \right]^2}$$

approx en linéarité

Les Resolveurs (mesure d'angle)



Schema Equivalent



Si V_s variable $\Rightarrow \vec{B}$ variable

$$\Rightarrow B = B_0 \sin \omega t$$

$\Rightarrow \theta$ variable de la station

$$\Rightarrow V_{\sin} \neq 0$$

$$\Rightarrow V_{\cos} \neq 0$$

$$\begin{cases} \theta_{\text{stat cos}} = B \cdot S \cos \theta & "B \cdot S" \end{cases}$$

$$\begin{cases} \theta_{\text{stat sin}} = B \cdot S \cdot \cos \left(\frac{\pi}{2} - \theta \right) = B \cdot S \sin \theta \end{cases}$$

Bien sur :

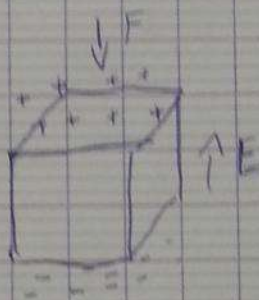
$$\begin{cases} V_{\cos} = \frac{d\theta_{\text{stat cos}}}{dt} = \sin B_0 \cos \omega t \cdot S \cos \theta \end{cases}$$

$$\begin{cases} V_{\sin} = \frac{d\theta_{\text{stat sin}}}{dt} = \sin B_0 S \cos \omega t \cdot \sin \theta \end{cases}$$

$$V_{\cos} \quad \text{---} \quad \frac{1 + \cos^2 \omega t}{2} \cdot \sin 2\theta \quad \xrightarrow{\text{moyenne}} \quad \sin 2\theta$$

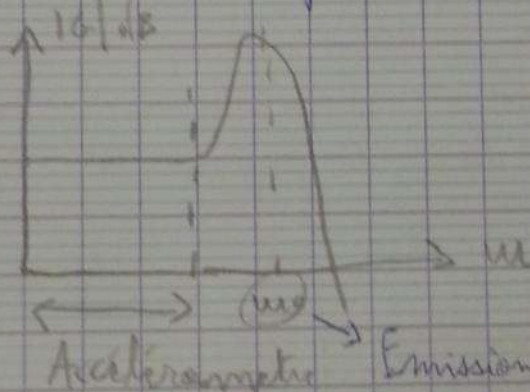
CAPTEURS DIEZOELECTRIQUE

1) Principe



$$Q = kF$$

App. d'une charge sous l'effet de F
 Réversibilité : Déformation suite à l'app. d'un champ E



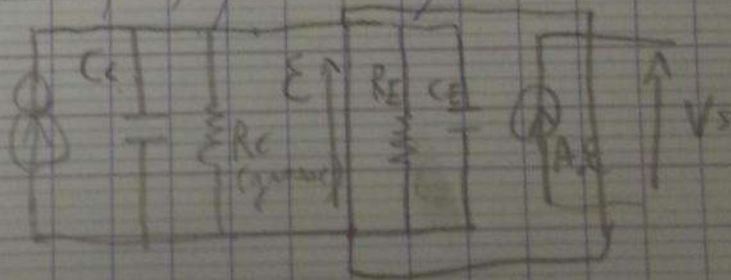
US

↓
Télémetrie

2) Circuit de conditionnement

↳ Modèle Électrique, position du probe

$$i = \frac{dQ}{dt}$$

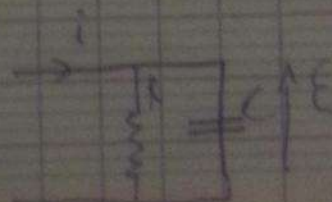


Ampli de tension

$$i = k \frac{dF}{dt}$$

$$I = jk \omega F$$

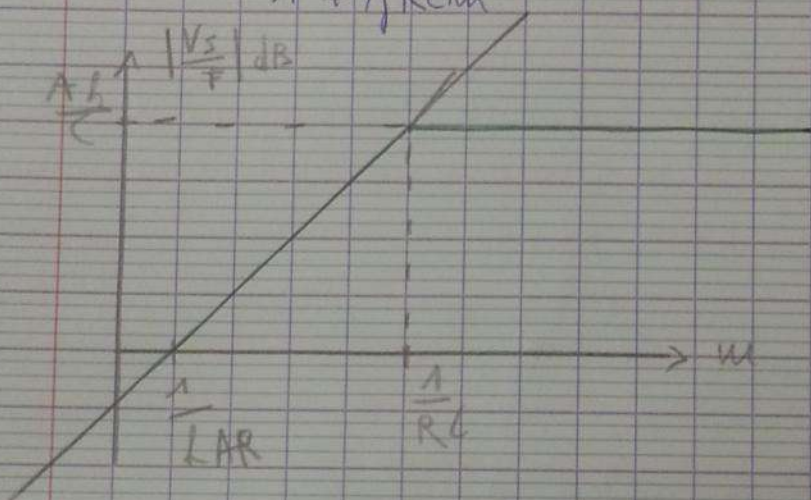
Modèle Entrée Ampli



$$V_s = A \cdot \varepsilon = G (R \parallel C) i$$

$$= A \cdot \frac{R}{R + \frac{1}{g_{cm}}} \cdot g_{cm} F$$

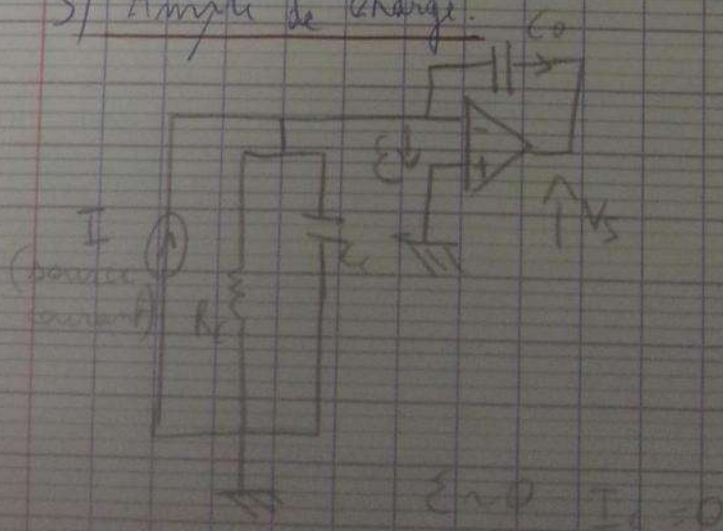
$$= \frac{A R \cdot g_{cm}}{1 + g_{cm} R} \cdot F$$



$\Rightarrow$ Dépend de la freq

$\Rightarrow$ ~~Ampli de Tension~~
 $\Downarrow$
 Ampli de charge

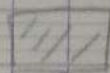
3) Ampli de charge:



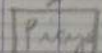
$$I = j \cdot C_m = j \cdot h \cdot m \cdot F$$

$$V_s = -\frac{h}{\epsilon_0} F : \text{Indépend. de } m$$

4) Application en accélérométrie



Masse sismique.

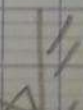


$$m_s = \sqrt{\frac{k}{\ddot{m}}} \quad \text{On travaille à } m < m_s$$

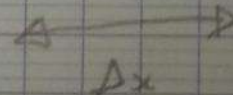
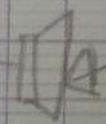
5) Applie en Télémétrie

1) Télémétrie par temps de vol

(E)



(R)



v : la vitesse de l'onde

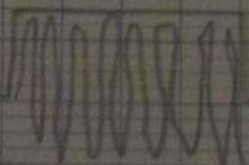
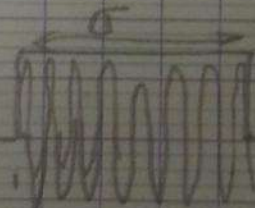
$$v = 350 \text{ m/s}$$

(onde acoustique)

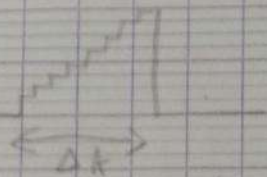
$$v = \frac{2 \Delta x}{\Delta t}$$

$$\Delta x = \frac{v \Delta t}{2}$$

(E)



choix de l'horloge du compteur



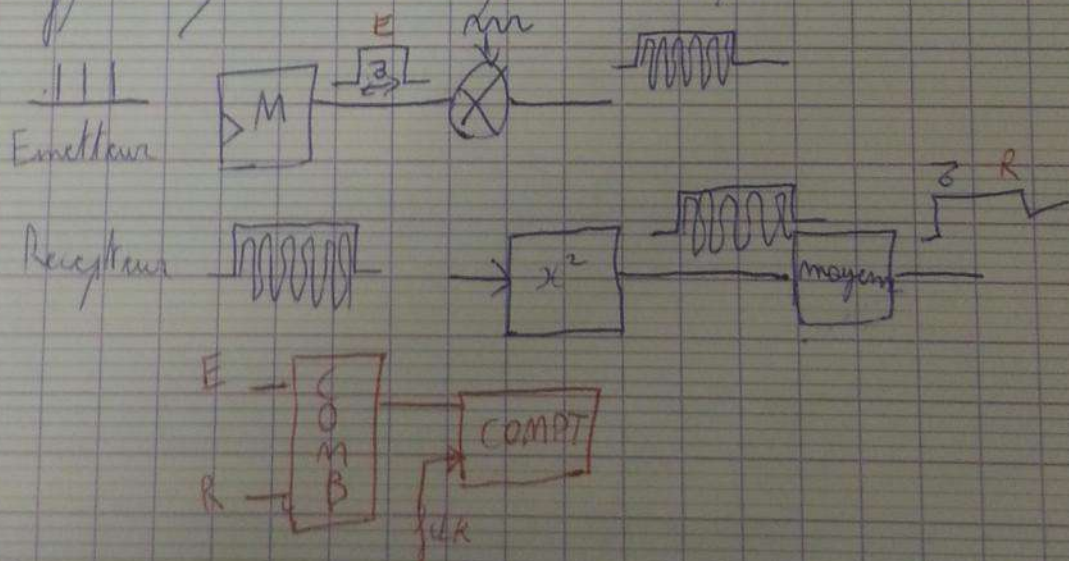
* Pour 1 précision de $1 \text{ cm} = 10^{-2} \text{ m}$
 $\Delta t_{\text{CLK}} = \frac{2 \Delta x}{v}$ période CLK

$$f_{\text{CLK}} = \frac{v}{2 \Delta x} = \frac{350}{2 \cdot 10^{-2}} = \frac{35000}{2} = 17500 \text{ Hz}$$

* Précision 1 dm = 0,1 m $\Rightarrow f_{\text{CLK}} = 1750 \text{ Hz}$

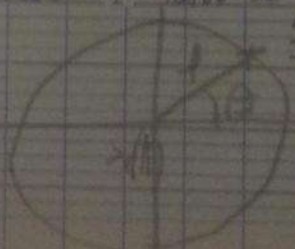
~~Directivité / optimisation~~

~~hypothèse : source ponctuelle / omnidirectionnelle~~



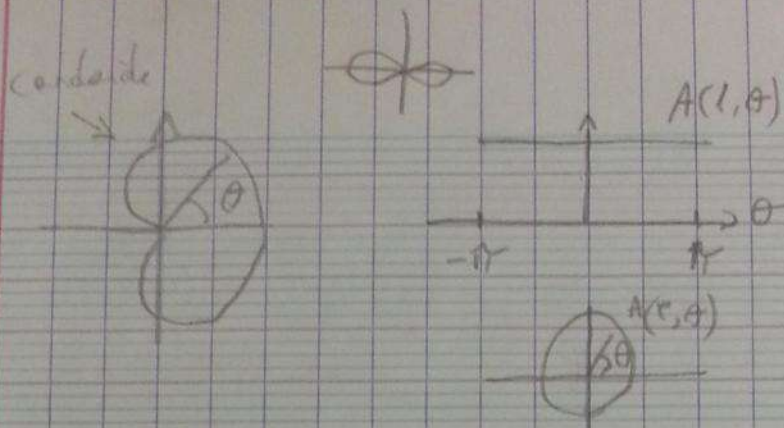
~~Directivité / optimisation~~

~~hypothèse : source ponctuelle / omnidirectionnelle~~

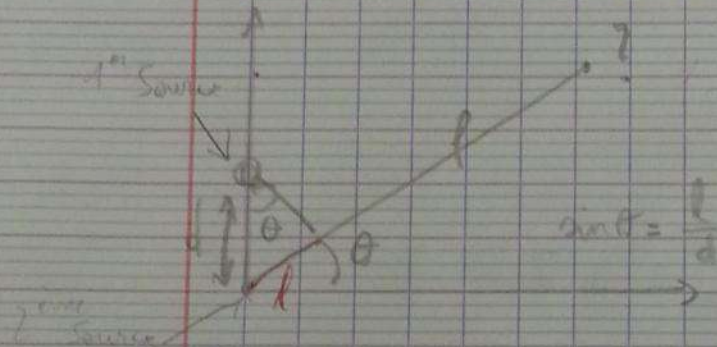


$$x(t, \theta) = A(r, \theta) \cdot \cos(\omega t + \phi)$$

$$A(r, \theta) = A(r) \quad \text{omnidirectionnelle}$$



1) Association de 2 sources



Examiner la résultante des 2 sources en champs lointain

$$x(t) = A(P) \cdot \cos(\omega t + \phi)$$

l'onde qui arrive en retard

$$x(t) = A(P) \cdot \cos(\omega(t + \frac{\phi}{\omega}))$$

$$\omega = 2\pi f \quad f = \frac{c}{\lambda}$$

$$\Delta t = \frac{\phi}{\omega} = \frac{l}{c} \quad l = d \sin \theta$$

$$l = \frac{c}{\omega} \phi$$

$$\Rightarrow \phi = \frac{l \omega}{c} = \frac{2\pi}{\lambda} d \sin \theta = 2\pi \frac{d}{\lambda} \sin \theta$$

$$\Rightarrow \boxed{\phi = 2\pi \frac{d}{\lambda} \sin \theta}$$

Notation complexe:

$$x(t) = A(\phi) \cos(\omega t + \phi)$$

$$\hookrightarrow A(\phi) e^{j(\omega t + \phi)}$$

$$A(\phi) e^{j\omega t}$$

$$X = A(\varphi) e^{i\varphi}$$

Resultante (Amplitude du signal reçu)

$$Y = A(\varphi) + A(\varphi) e^{i\varphi}$$

$$Y = A(\varphi) (1 + e^{i\varphi})$$

$$Y = A(\varphi) \cdot e^{i\frac{\varphi}{2}} (e^{i\frac{\varphi}{2}} + e^{-i\frac{\varphi}{2}})$$

$$Y = 2A(\varphi) e^{i\frac{\varphi}{2}} \cos \frac{\varphi}{2}$$

$$|Y| = 2|A(\varphi)| \cos \left(\pi \frac{d}{\lambda} \sin \theta \right)$$

Apparition une dépendance λ en θ

* Zones claires

$$\cos \left(\pi \frac{d}{\lambda} \sin \theta \right) = \pm 1$$

$$\Rightarrow \pi \frac{d}{\lambda} \sin \theta = m \pi$$

$$\Rightarrow \sin \theta = m \frac{\lambda}{d} \quad m \in \mathbb{Z}$$

* Zones obscures

$$\cos \left(\pi \frac{d}{\lambda} \sin \theta \right) = 0$$

$$\Rightarrow \pi \frac{d}{\lambda} \sin \theta = (2m+1) \frac{\pi}{2}$$

$$\Rightarrow \boxed{\sin \theta = (m + \frac{1}{2}) \frac{\lambda}{d}}$$

Exemple de rayonnement $d = \lambda$

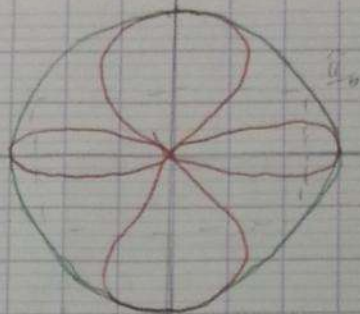
* Zones claires : $\sin \theta = m = 0, \pm 1$

$$\Rightarrow \begin{cases} \theta = 0 \text{ ou } \pi \\ \theta = \pm \frac{\pi}{2} \end{cases}$$

* Zones obscures : $\sin \theta = m + \frac{1}{2}$

$$\Rightarrow \sin \theta = \pm \frac{1}{2} \quad \theta = \pm \frac{\pi}{6}, \pm \frac{5\pi}{6}$$

2 sources :
 + directivité
 - portée
 + sensibilité



1 source :
 - sensibilité

On ne connaît pas la largeur (portée)

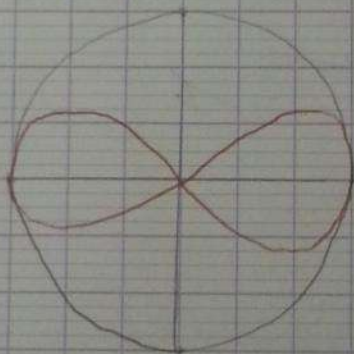
Exemple - $d = \frac{\lambda}{2}$

* Zones claires - $\sin \theta = 2m$
 $\Rightarrow \theta = 0$ ou π

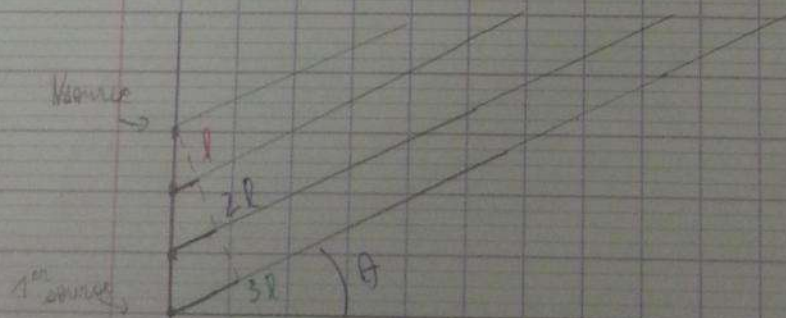
* Zones obscures - $\sin \theta = 2m + 1$

$m = 0 \Rightarrow \sin \theta = \pm 1 \quad \theta = \pm \frac{\pi}{2}$

$m = -1 \Rightarrow \sin \theta = \pm 1 \quad \theta = \pm \frac{\pi}{2}$



* Association de N sources



Resultante :

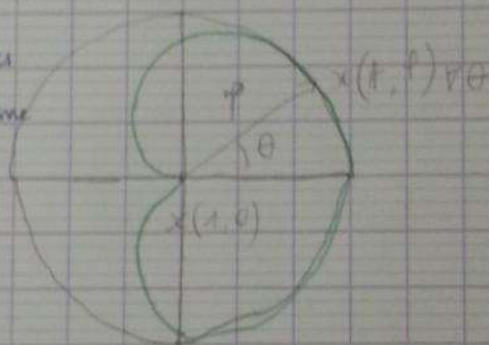
$$Y = A(\varphi) \left(1 + e^{j\varphi} + e^{2j\varphi} + e^{3j\varphi} + \dots + e^{(N-1)j\varphi} \right)$$

$$Y = A(p) \frac{1 - e^{-iNp}}{1 - e^{-ip}} = A(p) \frac{e^{iNp/2}}{e^{ip/2}} \frac{\sin(Np/2)}{\sin(p/2)}$$

$$|Y| = |A(p)| \cdot \frac{\sin(Np/2)}{\sin(p/2)}$$

* Source CARDIOÏDE $\Rightarrow$ l'équation: $A(p, \theta) = a(p) \cdot \frac{1 + \cos \theta}{2}$

Les deux sources
émettent comme
ça



Notation complexe:

$$X(p, \theta) = A(p, \theta) e^{ip}$$

$$|Y| = |A(p)| \underbrace{\cos\left(\frac{Np}{2} \sin \theta\right)}_{\text{déjà fait}} \underbrace{(1 + \cos \theta)}_{\text{nouveau}}$$

* Zone obscure:

$$(1 + \cos \theta) = 0$$

$$\theta = \pi$$

(+)

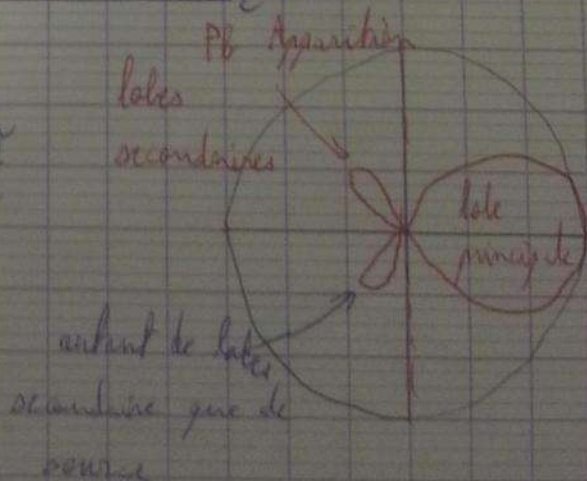
$$\text{Déjà fait} = \sin \theta = \left(n + \frac{1}{2}\right) \frac{\lambda}{d}$$

Exemple de rayonnement $d = \frac{\lambda}{2}$

zone obscure:

$$\theta = \pi$$

$$\sin \theta = \left(2n + 1\right) = \pm \frac{\pi}{2}$$



Cas de $N = 6$ sources cohérentes
 $d = \frac{\lambda}{2}$

zones d'ombres :

$$* 1 + \cos \theta = 0 \quad \theta = \pi$$

$$* \sin(N \pi \frac{d}{\lambda} \sin \theta) = 0$$

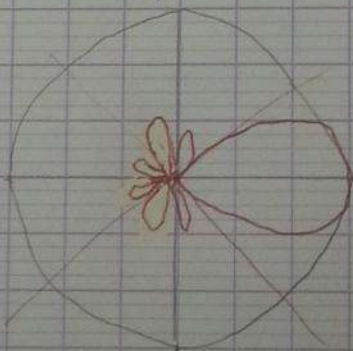
$$\sin(2\pi \sin \theta) = 0$$

$$2\pi \sin \theta = m\pi$$

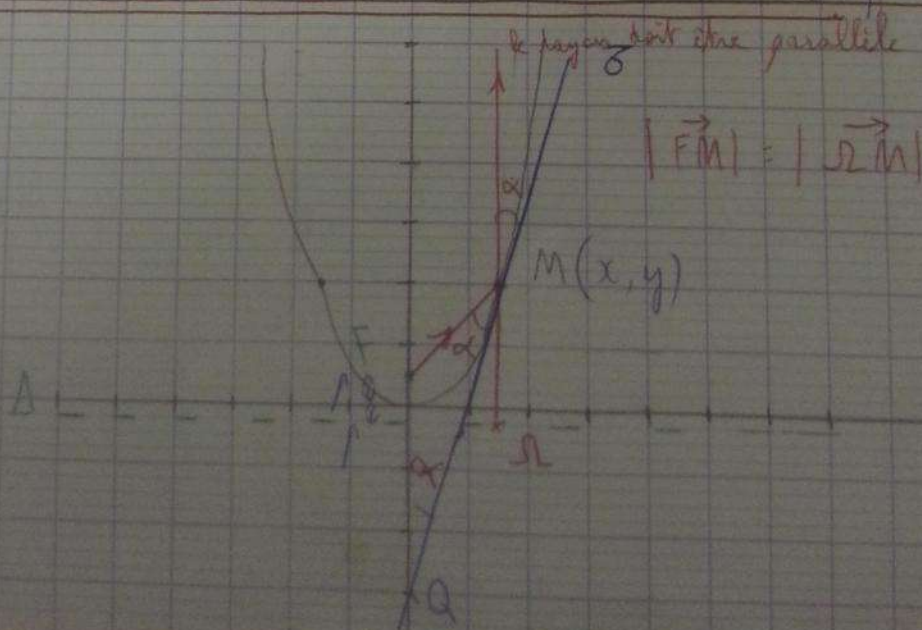
$$\sin \theta = \frac{m}{2} \quad m \in \mathbb{Z}$$

$$\sin \theta = 0, \pm \frac{1}{2}, \pm 1$$

$$\theta = 0, \pi, \pm \frac{\pi}{6}, \pm \frac{5\pi}{6}, \pm \frac{\pi}{2}$$



OPTIMISATION / REFLECTEUR PARABOLIQUE



$\mathcal{P}$: Ensemble des pts M équidistant à 1 droite Δ et à 1 pt F

Directrice

Foyer

1) Equation cartésienne

$$\vec{OF} = \begin{pmatrix} 0 \\ p \end{pmatrix} \quad \vec{O\Omega} = \begin{pmatrix} 0 \\ -p \end{pmatrix}$$

$$\vec{\Omega M} = \begin{pmatrix} 0 \\ y+p \end{pmatrix} \quad \vec{OM} = \vec{OF} + \vec{FM}$$

$$\vec{FM} = \vec{OM} - \vec{OF}$$

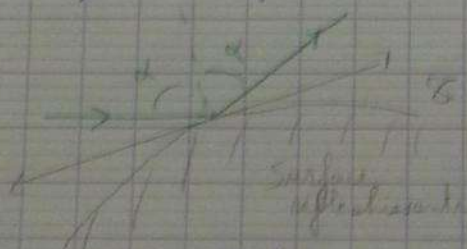
$$\vec{FM} = \begin{pmatrix} x \\ y-p \end{pmatrix}$$

$$|\vec{FM}| = |\vec{\Omega M}| \Rightarrow x^2 + (y-p)^2 = (y+p)^2$$

$$x^2 + y^2 - 2yp + p^2 = y^2 + p^2 + 2yp$$

$$x^2 = 4yp \Rightarrow \boxed{y = \frac{x^2}{4p}}$$

2) Propriété réfléchissante



On montre que tout rayon émanant de F sort de la $\mathcal{P}$ parallèlement à Δ
Si vrai alors $FM = FQ$

(E) Soit $M(x_0, y_0)$

$$f(x) = y_0 + f'(x_0) \cdot (x - x_0)$$

$$= y_0 + \frac{2x_0}{4p} \cdot (x - x_0)$$

$$f(x) = y_0 + \frac{x_0}{2p} (x - x_0)$$

$$\vec{OQ} = \begin{pmatrix} 0 \\ y_0 - \frac{x_0^2}{2p} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{x_0^2}{4p} \end{pmatrix}$$

$$\Rightarrow \vec{FQ} = \begin{pmatrix} 0 \\ -1 - \frac{x_0^2}{4p} \end{pmatrix}$$

$$FQ = p + \frac{x_0^2}{4p}$$

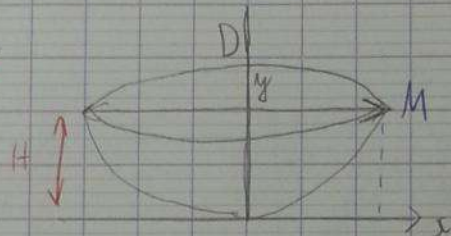
$$\vec{FM} = \vec{OM} - \vec{OF}$$

$$= \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \begin{pmatrix} 0 \\ p \end{pmatrix}$$

$$\begin{aligned}
 &= \begin{pmatrix} x_0 \\ y_0 - p \end{pmatrix} \\
 FM^2 &= x_0^2 + (y_0 - p)^2 \\
 &= x_0^2 + y_0^2 + p^2 - 2y_0 p \\
 &= x_0^2 + \left(\frac{x_0^2}{4p}\right) + p^2 - \frac{x_0^2}{2} \\
 \boxed{FM^2} &= \frac{1}{2} x_0^2 + \left(\frac{x_0^2}{4p}\right) + p^2
 \end{aligned}$$

III Q4

1



$$M = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} D/2 \\ H \end{pmatrix} \quad \text{or} \quad y = \frac{x^2}{4p}$$

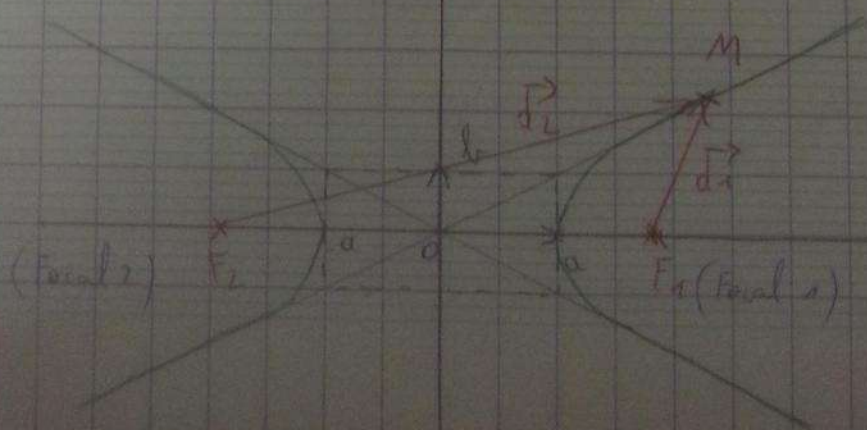
$$H = \frac{(D/2)^2}{4p} = \frac{D^2}{16p}$$

$$p = \frac{D^2}{16H} = \frac{160^2}{16 \times 10} = 10 \text{ cm}$$

pour $H = 20 \text{ cm} \rightarrow p = 5 \text{ cm}$

TELEMETRIE HYPERBOLIQUE

1) Définition géométrique d'une hyperbole H



H : Ensemble des pts $M \begin{pmatrix} x \\ y \end{pmatrix}$ tel que :

$$\begin{aligned} \vec{OF_1 M} - \vec{OF_2 M} &= \pm 2a \text{ (constante)} \\ |\vec{d_1}| - |\vec{d_2}| &= \pm 2a \end{aligned}$$

2) Equation cartésienne

$$\vec{OM} = \vec{OF_1} + \vec{d_1} = \vec{OF_2} + \vec{d_2}$$

$$\begin{cases} \vec{d_1} = \vec{OM} - \vec{OF_1} \\ \vec{d_2} = \vec{OM} - \vec{OF_2} \end{cases}$$

$$M \begin{pmatrix} x \\ y \end{pmatrix} \quad \vec{d_1} = \begin{pmatrix} x-c \\ y \end{pmatrix} \quad \vec{d_2} = \begin{pmatrix} x+c \\ y \end{pmatrix}$$

$$\text{Au sommet : } \begin{cases} d_2 = c+a \\ d_1 = c-a \end{cases}$$

$$\text{On a bien } d_2 - d_1 = 2a \\ d_2 = d_1 + 2a$$

$$\begin{aligned} d_2^2 &= (x+c)^2 + y^2 = (d_1 + 2a)^2 = d_1^2 + 4a^2 + 4ad_1 \\ (x+c)^2 + y^2 &= (x-c)^2 + y^2 + 4a^2 + 4a\sqrt{(x-c)^2 + y^2} \end{aligned}$$

$$\cancel{x^2} + \cancel{y^2} + 2xc + y^2 = \cancel{x^2} + \cancel{y^2} - 2xc + y^2 + 4a^2 + 4a\sqrt{(x-c)^2 + y^2}$$

$$xc = a^2 + a\sqrt{(x-c)^2 + y^2}$$

$$(xc - a^2)^2 = (a\sqrt{(x-c)^2 + y^2})^2$$

$$x^2c^2 + a^4 - 2a^2xc = a^2[(x-c)^2 + y^2]$$

$$x^2c^2 + a^4 - 2a^2xc = a^2(x^2 - 2xc + c^2 + y^2)$$

$$x^2c^2 + a^4 = a^2x^2 + a^2c^2 + a^2y^2$$

$$x^2(c^2 - a^2) - a^2y^2 = a^2c^2 - a^4$$

$$x^2\left(\frac{c^2 - a^2}{a^2}\right) - y^2 = c^2 - a^2$$

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

$$\text{Avec } \begin{cases} b^2 = c^2 - a^2 \\ c^2 = a^2 + b^2 \end{cases}$$

Complete Asymptote $(x) \rightarrow \infty$
 $\frac{x^2}{a^2} - \frac{y^2}{b^2} \approx 0 \Rightarrow \frac{y}{x} = \pm \frac{b}{a}$

$\Rightarrow$ Les droites de pentes $\pm \frac{b}{a}$ sont Asymptotes Obliques.

3) Application en géopositionnement: LORAN

$M(x, y)$: Position d'un bateau

F_1 et F_2 : Position de deux stations radio émettant des impulsions synchrones

t_2 : le temps de propagation de l'onde S₂ de la zone station

$$V = \frac{a}{t} ; \quad k_2 = \frac{dz}{v} \quad k_1 = \frac{da}{v}$$

$\Delta t = \frac{dz - dv}{c} = \frac{2a}{c} \Rightarrow$ On peut dire que le bateau se trouve sur une hyperbole képlérienne $\left| \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \right|$ avec $a = \frac{1}{2} v \cdot \Delta t$

$C = \frac{S_1 S_2}{2}$ Distances entre les stations

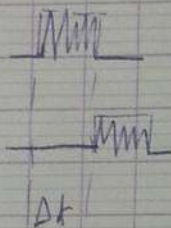
$$S_1 + S_2 = 100 \text{ km}$$

$$\Delta t = 200 \mu s$$

Lieux des M / $|\vec{d}_2| - |\vec{d}_1| = 2a : \text{cte}$
 $\vec{d}_1 = \vec{S}_1 M$
 $\vec{d}_2 = \vec{S}_2 M$ $|\vec{S}_1 \vec{S}_2| = 2c$

↳ TD Géoposit

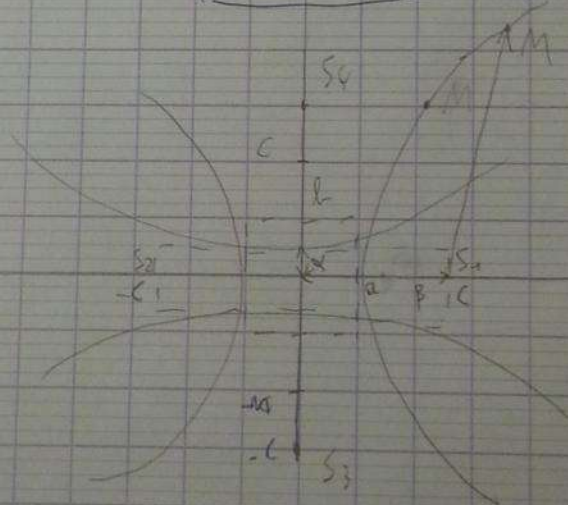
$$\begin{cases} t_2 = \frac{d_2}{v} \\ t_1 = \frac{d_1}{v} \end{cases}$$



$$\Delta t = \frac{d_2 - d_1}{v} = \frac{2a}{v}$$

$d_2 - d_1 = \text{cte} \rightarrow \text{lien } M$

$$\boxed{M \in H \quad R = \frac{v \Delta t}{2}} \\ c = \frac{S_1 S_2}{2}$$



$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$

$$a^2 + b^2 = c^2$$

Asymptote de pente $\frac{b}{a}$

En permettant x et y

$$\left(\frac{y}{b}\right)^2 - \left(\frac{x}{a}\right)^2 = 1$$

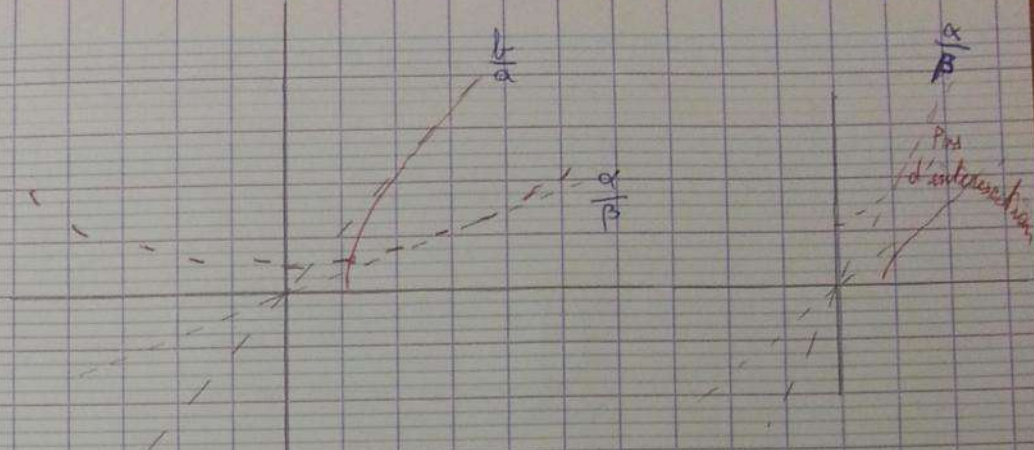
Asymptote de pente $\frac{a}{b}$

$$a^2 + b^2 = c^2$$

$$a = v \Delta t$$

Δt : Retard $S_3 S_4$

$$c = \frac{S_3 S_4}{2}$$



Limite $\frac{l}{a} > \frac{a}{B}$ $a < Bl$ $(la)^2 < (Bl)^2 = (c^2 - a^2)/c^2$
 $(la)^2 < c^2 + (la)^2 - c^2(a^2 + a^2) \Rightarrow c^2 - (a^2 + a^2) > 0$ 92

1) Limite $\frac{v \Delta t}{2} < c$ $\Delta t < \frac{2c}{v}$
 $\Delta t < \frac{2c}{v}$

$\frac{2c}{v} = \frac{10^8}{3 \cdot 10^8} = 0,333 \cdot 10^3$
 $= 330 \mu s$

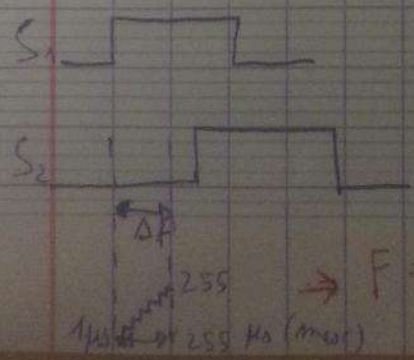
Pas de Bateman : * $\Delta t = 200 \mu s \Rightarrow a = \frac{v \Delta t}{2} = \frac{100 \cdot 10^6 \cdot 3 \cdot 10^8}{2} = 3 \cdot 10^6 = 30 km$
 $b^2 = c^2 - a^2 = (50)^2 - (30)^2 = (50-30)(50+30)$
 $= 20 \times 80 = 1600$

$b = 40 km$

* $\Delta t = 250 \mu s \Rightarrow 125 \cdot 10^6 \cdot 3 \cdot 10^8 = 375 km$

$b^2 = c^2 - a^2 \dots$ $b = 33,1 km$

$x = 78 km$
 $y = 96 km$

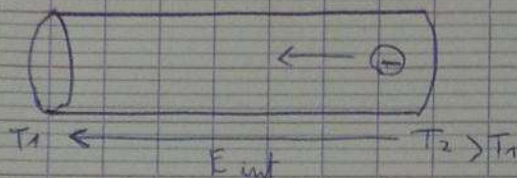


$\rightarrow F = 1 MHz$

$\frac{v \Delta t}{2}$
 Encre $\frac{10^6 \cdot 3 \cdot 10^8}{2} = 150 m$

EFFET THERMOELECTRIQUE

1) Effet Seebeck



$$\vec{E}_{int} \propto \vec{\nabla} T$$

$$\Theta_A dT = dV$$

$$E_{int} = -\Theta_A \frac{dT}{dx} = -\frac{dV}{dx}$$

$$dV = \Theta_A(T) dT$$

$\frac{mV}{^\circ K}$ dépend de T

Dans tout ce qui suit on

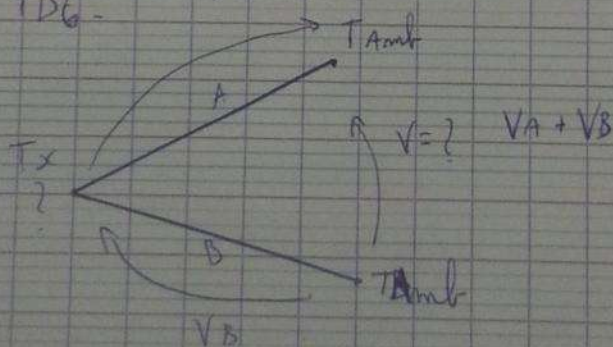
suppose $\Theta_A(T) = \Theta_A$: cste

$$\Delta V = \Theta_A (T_2 - T_1)$$

"Approx. lin"

2) Thermocouple

TDB -



$$V_A = \Theta_A (T_A - T_x)$$

$$V_B = \Theta_B (T_x - T_A)$$

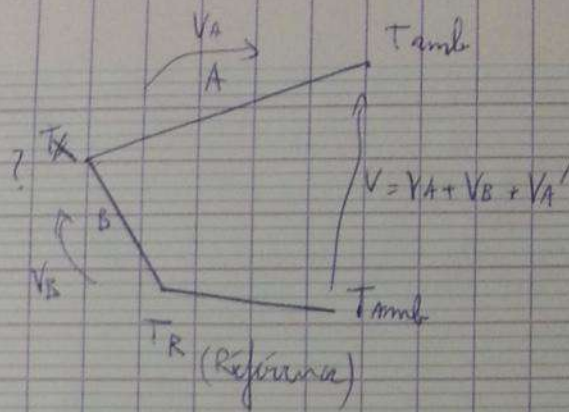
$$V = (\Theta_B - \Theta_A) (T_x - T_A)$$

Θ_{BA}

$$V = \Theta_{BA} (T_x - T_A)$$

$$\Theta_{BA} = \Theta_B - \Theta_A$$

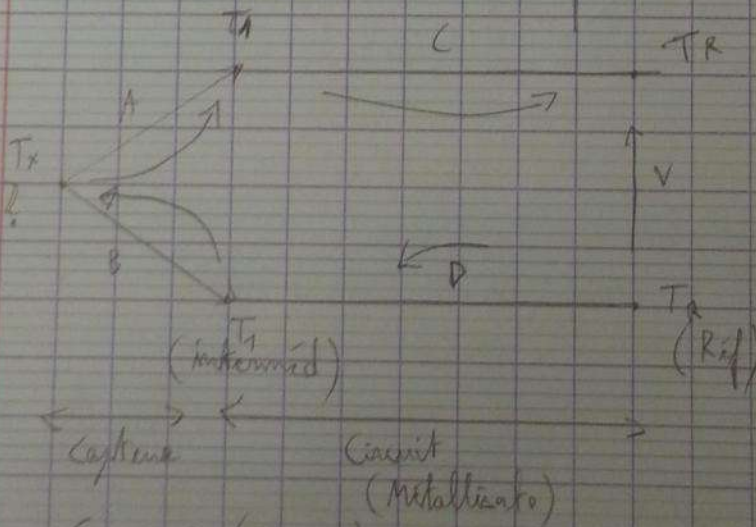
$\downarrow$ T_{amb}



$$\begin{cases} V_A = \theta_A (T_A - T_x) \\ V_B = \theta_B (T_x - T_R) \end{cases}$$

$$\begin{cases} V_{A'} = \theta_A (T_R - T_A) \\ V = \theta_B (T_x - T_R) - \theta_A (T_x - T_R) \\ V = \theta_{BA} (T_x - T_R) \end{cases}$$

Type	Thermo-couple	θ°	Scale (mV)
T	Cu-Fe - Constantan	-	10
J	Fe - Ni	-200°C to 370°C	5
K	Chromel - Alumel	270°C to 1250°C	50
E	Chromel - Constantan	270°C to 270°C	50



$$\begin{cases} V_C = \theta_C (T_R - T_x) \\ V_A = \theta_A (T_A - T_x) \\ V_B = \theta_B (T_x - T_R) \\ V_D = \theta_D (T_A - T_R) \end{cases}$$

$$V = \theta_{CD} (T_R - T_x) + \theta_{BA} (T_x - T_R)$$

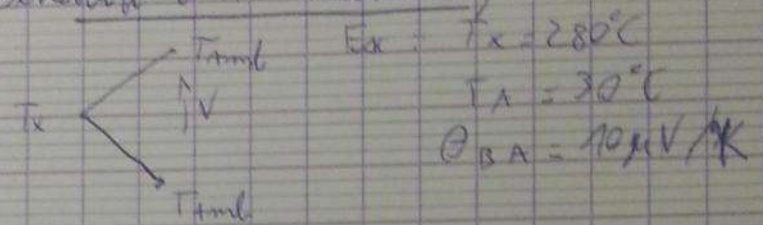
$$\text{On next } \theta_{BA} (T_x - T_R)$$

$$\Rightarrow \text{Interm} = \theta_{CD} (T_R - T_x) + \theta_{BA} (T_R - T_x) = 0$$

$$\Rightarrow \theta_{CD} (T_R - T_x) - \theta_{AB} (T_R - T_x) = 0$$

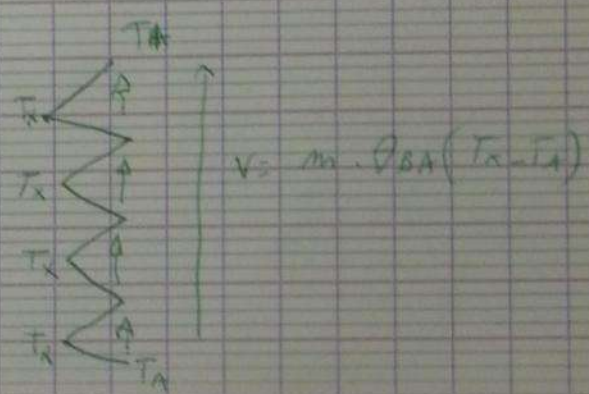
Il faut CD proche AB
 $\Theta_{CD} = \Theta_{AB}$

Sensibilité d'un thermocouple



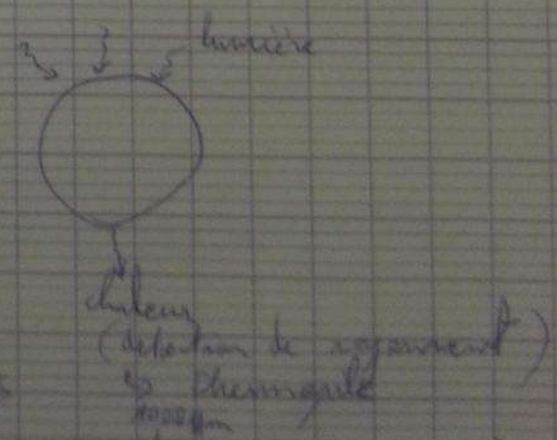
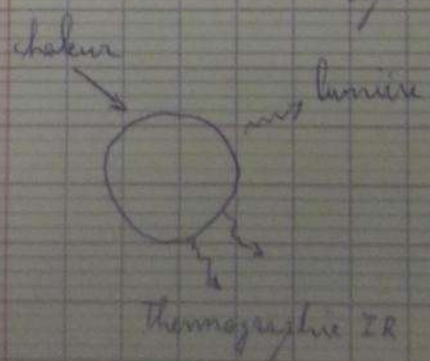
$$V = \Theta_{BA} (T_x - T_A) = 10 \cdot 10^{-6} \times 250 = 2,5 \text{ mV (petit)}$$

Technique :



PYROMETRIE - THERMOGRAPHIE Infrarouge

- ↳ Mesure T° très élevée
- ↳ ... sans contact
- ↳ ... corps en mouvement



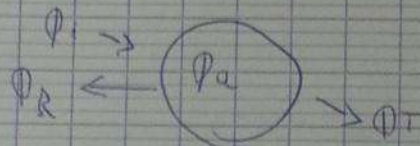
0,35 μm	0,35	1,5	15	
Visible	IR	Infra rouge moyen	Infra rouge lointain	onde

1) Définition : Radiance

* $R(\lambda, T)$ Radiance spectrale.
= Puissance Rayonnée par unité de surface et pour longueur d'onde à une T° donnée

* $R(T) = \int_0^\infty R(\lambda, T) d\lambda$: Radiance totale

* Calc de transmission, l'absorption, réflexion



$$P_i = P_a + P_r + P_t$$

$$1 = \alpha + \tau + \rho$$

* Modèle Théorique d'un corps Noir (CN)
Absorbe toute la lumière

$$\alpha = 1 \quad \tau = \rho = 0$$

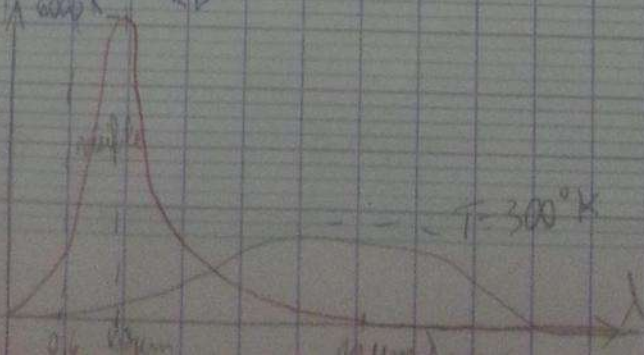
$\Rightarrow$ 1 CN éclairé $\Rightarrow T^\circ \uparrow$
Rayonnement du CN

La radiance spec d'un CN :

$$R^\circ(\lambda, T) = \frac{c_1}{\lambda^5} \cdot \frac{1}{\exp\left(\frac{c_2}{\lambda T}\right) - 1}$$

$$\begin{cases} c_1 = 2\pi^5 h c^2 = 3,74 \cdot 10^8 \text{ W m}^2 \\ c_2 = hc = 1,98 \cdot 10^{-25} \text{ mK} \end{cases}$$

$$R^\circ(\lambda, T) \propto \frac{1}{\lambda^5 \exp\left(\frac{c_2}{\lambda T}\right)}$$



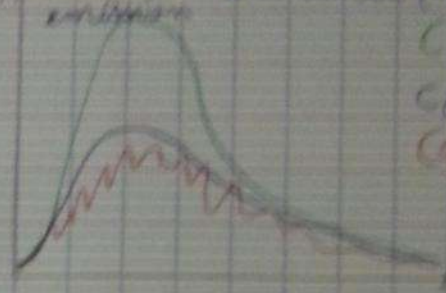
$$\lambda_{max} = \frac{b}{T} \approx \frac{2.9 \cdot 10^3}{T}$$

$$R^*(T) = \int R^*(\lambda, T) d\lambda = \sigma T^4 \text{ W/m}^2 \text{ Loi de Stefan}$$

$$\sigma = 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \text{ constante de Stefan}$$

$R^*(\lambda, T)$: émission de la lamelle (on chauffe les rayons)

Modeler approximer
Rayleigh
 $\lambda T < 2.82$
 $R^*(\lambda, T) = \frac{15}{\lambda^5} \frac{1}{e^{\frac{1}{\lambda T}} - 1}$



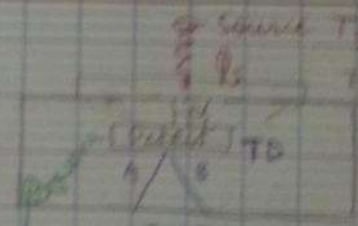
CN
CG
CQ : corps quelconque

λ : de la lumière IR, UV...

When $\lambda T \gg 2.82$ Pouvoir Emissif
 $R(\lambda, T) \approx \frac{15}{\lambda^5} \exp\left(-\frac{1}{\lambda T}\right)$

$$E(\lambda, T) = \frac{R(\lambda, T)}{R^*(\lambda, T)}$$

2) Technique Anatomique d'un thermographe



α : angle d'ouverture
 $\alpha = 30^\circ$
(pour amplifier)
section réduite

$$Q_s = \sigma T_s^4 S_s$$

$$Q_D = \sigma T_D^4 S_D$$

Pour caractériser $V_s = \frac{1}{S} (Q_s - Q_D) = \sigma S_D (T_s^4 - T_D^4)$

$\frac{1}{S}$: sensibilité

$$V_s = 2\sigma \alpha (T_D - T_A)$$

