

There a big correlation between the results on test A and the results on test B. We see that the X follow the linear regression Y .

$$3 - Y = \hat{\beta}_0 + \hat{\beta}_1 X + \varepsilon_i$$

$$\Rightarrow Y = 4,23 + 1,043 \times 5$$

$$\underline{Y = 9,445}$$

$$\text{Or : } \underline{Y = X + R^2 \times X = 9,55}$$

If a candidate obtains 5 on the test A, he should obtain average 9 on test B.

Exercise 1: 4/4

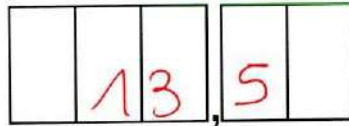
1. B. Yes
2. A. Always
3. B.
4. A.

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MATIÈRE Statistics For Finance

Exercice 2: 3/6

1 - μ ? $n = 100$, $\bar{X} = 48$ cl 90% and 95% IC.

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \Rightarrow \text{IC: } \bar{X} - u \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + u \frac{\sigma}{\sqrt{n}}$$

For 90% IC: $u = 1,645$

95%: $2,96$

because $\alpha_1 = 100 - 90 = 10\%$

$\Rightarrow \alpha_2 = 100 - 95 = 5\%$

$$\Phi_1\left(1 - \frac{\alpha_1}{2}\right) = \Phi\left(1 - 2,5\%\right) = 0,975$$

$$\Phi_2\left(1 - \frac{\alpha_2}{2}\right) = \Phi\left(1 - 5\%\right) = 0,95$$

Donc pour: 90%:

$$\begin{aligned} \text{IC} &= 48 - 1,64 \times \frac{5}{10} \leq \mu \leq 48 + 1,64 \times \frac{5}{10} \\ &= 48 - 0,82 \leq \mu \leq 48 + 0,82 \\ &= \boxed{47,18 \leq \mu \leq 48,82} \end{aligned}$$

For 95% IC:

$$\begin{aligned} &48 - 1,96 \times \frac{5}{10} \leq \mu \leq 48 + 1,96 \times \frac{5}{10} \\ &= \boxed{47,02 \leq \mu \leq 48,98} \end{aligned}$$

2. Suppose the sample size is unknown. What sample size is required to estimate the average contents to be within 0.5 cl at the 95% confidence level?
3. A sample of 36 bottles yields to an average content of 48.5. Test the hypothesis that the average content per bottle is 50 at the 5% significance level.

Exercise 3. Suppose we're looking at a two-candidate mayoral race in Providence.

Candidate A is declared the winner with 55 percent of the vote (i.e. $\hat{p} = 0.55$). However, Candidate B is suspicious of these results. Having his group of close friends take a random sample of 17 voters from Providence, he finds that 7 voted for Candidate A while 10 voted for him.

On the basis of this study performed at the $\alpha = 0.05$ level of significance, should Candidate B demand a recount?

Formulate the null and alternative hypothesis and perform the test in order to respond to this question.

Exercise 4. 12 people are enrolled in a training. At the beginning of the training, these trainees take a test A rated out of 20. At the end of the training, they take test B with a same level as the test A. The results are given in the following table :

Test A	3	4	6	7	9	10	9	11	12	13
Test B	8	9	10	13	15	14	13	16	13	19

1. Represent the scatter plot. Calculate the least squares line for the model ($Y = \beta_0 + \beta_1 X + \varepsilon_i$) Where Y is Test B and X is Test A.
2. Calculate the coefficient of determination R^2 and comment it.
3. If a candidate obtained 5 on the test A what should he obtain for the test B?

$$P(-1,66 < Z) \Leftrightarrow P(1,66 < Z) = 1 - P(1,66 < Z) \\ = 1 - 0,9515 = 0,0485 < \alpha = 0,05$$

We reject the hypothesis H_0 .

The candidate B should demand recount

Exercise 4: ~~5/6~~

$$1 - \text{Cov}(X, Y) = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}) : \boxed{\bar{X} = 8,4, \bar{Y} = 13}$$

$$\begin{array}{l} \bar{A} = 8,4, \quad \bar{B} = 13 \quad \sum (A_i - \bar{A}) = 0 \Rightarrow \text{Cov}(A, B) = 0 \\ \Rightarrow \hat{\beta}_1 = 0 \quad \Rightarrow Y = \hat{\beta}_0 \Rightarrow \boxed{\hat{\beta}_0 = 13} \\ \boxed{\bar{Y} = 13} \quad \boxed{\bar{X} = 8,4} \end{array}$$

$$\text{Cov}(X, Y) = 10,48$$

$$\text{VAR}(X) = 10,04$$

$$\hat{\beta}_1 = \frac{\text{Cov}(X, Y)}{\text{VAR}(X)} = \frac{10,48}{10,04} = \boxed{1,043}$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 13 - 8,4 \times 1,043 = \boxed{4,23}$$

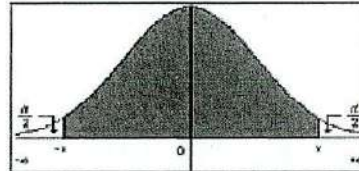
$$Y = 4,23 + 1,043 \times 8,4$$

$$2 - R^2 = \frac{\text{Cov}(X, Y)^2}{\text{VAR}(X) \times \text{VAR}(Y)} = \frac{10,48^2}{10,04 \times 12} = \frac{109,83}{120,48} = 0,91$$

$$\text{Var}(Y) = 12$$

$$\boxed{R^2 = 0,91}$$

Critical Values of the Student's t-distribution



α	1	0,8	0,6	0,4	0,2	0,1	0,05	0,02	0,01	0,002	0,001
$1 - \alpha$	0	0,2	0,4	0,6	0,8	0,9	0,95	0,98	0,99	0,998	0,999
$v = \text{ddl}$											
1	0,0000	0,3249	0,7265	1,3764	3,0777	6,3137	12,706	31,821	63,656	318,29	636,58
2	0,0000	0,2887	0,6172	1,0607	1,8856	2,9200	4,3027	6,9645	9,9250	22,328	31,600
3	0,0000	0,2767	0,5844	0,9785	1,6377	2,3534	3,1824	4,5407	5,8408	10,214	12,924
4	0,0000	0,2707	0,5686	0,9410	1,5332	2,1318	2,7765	3,7469	4,6041	7,1729	8,6101
5	0,0000	0,2672	0,5594	0,9195	1,4759	2,0150	2,5706	3,3649	4,0321	5,8935	6,8685
6	0,0000	0,2648	0,5534	0,9057	1,4398	1,9432	2,4469	3,1427	3,7074	5,2075	5,9587
7	0,0000	0,2632	0,5491	0,8960	1,4149	1,8946	2,3646	2,9979	3,4995	4,7853	5,4081
8	0,0000	0,2619	0,5459	0,8889	1,3968	1,8595	2,3060	2,8965	3,3554	4,5008	5,0414
9	0,0000	0,2610	0,5435	0,8834	1,3830	1,8331	2,2622	2,8214	3,2498	4,2969	4,7809
10	0,0000	0,2602	0,5415	0,8791	1,3722	1,8125	2,2281	2,7638	3,1693	4,1437	4,5868
11	0,0000	0,2596	0,5399	0,8755	1,3634	1,7959	2,2010	2,7181	3,1058	4,0248	4,4369
12	0,0000	0,2590	0,5386	0,8726	1,3562	1,7823	2,1788	2,6810	3,0545	3,9296	4,3178
13	0,0000	0,2586	0,5375	0,8702	1,3502	1,7709	2,1604	2,6503	3,0123	3,8520	4,2209
14	0,0000	0,2582	0,5366	0,8681	1,3450	1,7613	2,1448	2,6245	2,9768	3,7874	4,1403
15	0,0000	0,2579	0,5357	0,8662	1,3406	1,7531	2,1315	2,6025	2,9467	3,7329	4,0728
16	0,0000	0,2576	0,5350	0,8647	1,3368	1,7459	2,1199	2,5835	2,9208	3,6861	4,0149
17	0,0000	0,2573	0,5344	0,8633	1,3334	1,7396	2,1098	2,5669	2,8982	3,6458	3,9651
18	0,0000	0,2571	0,5338	0,8620	1,3304	1,7341	2,1009	2,5524	2,8784	3,6105	3,9217
19	0,0000	0,2569	0,5333	0,8610	1,3277	1,7291	2,0930	2,5395	2,8609	3,5793	3,8833
20	0,0000	0,2567	0,5329	0,8600	1,3253	1,7247	2,0860	2,5280	2,8453	3,5518	3,8496
21	0,0000	0,2566	0,5325	0,8591	1,3232	1,7207	2,0796	2,5176	2,8314	3,5271	3,8193
22	0,0000	0,2564	0,5321	0,8583	1,3212	1,7171	2,0739	2,5083	2,8188	3,5050	3,7922
23	0,0000	0,2563	0,5317	0,8575	1,3195	1,7139	2,0687	2,4999	2,8073	3,4850	3,7676
24	0,0000	0,2562	0,5314	0,8569	1,3178	1,7109	2,0639	2,4922	2,7970	3,4668	3,7454
25	0,0000	0,2561	0,5312	0,8562	1,3163	1,7081	2,0595	2,4851	2,7874	3,4502	3,7251
26	0,0000	0,2560	0,5309	0,8557	1,3150	1,7056	2,0555	2,4786	2,7787	3,4350	3,7067
27	0,0000	0,2559	0,5306	0,8551	1,3137	1,7033	2,0518	2,4727	2,7707	3,4210	3,6895
28	0,0000	0,2558	0,5304	0,8546	1,3125	1,7011	2,0484	2,4671	2,7633	3,4082	3,6739
29	0,0000	0,2557	0,5302	0,8542	1,3114	1,6991	2,0452	2,4620	2,7564	3,3963	3,6595
30	0,0000	0,2556	0,5300	0,8538	1,3104	1,6973	2,0423	2,4573	2,7500	3,3852	3,6460
40	0,0000	0,2550	0,5286	0,8507	1,3031	1,6839	2,0211	2,4233	2,7045	3,3069	3,5510
50	0,0000	0,2547	0,5278	0,8489	1,2987	1,6759	2,0086	2,4033	2,6778	3,2614	3,4960
60	0,0000	0,2545	0,5272	0,8477	1,2958	1,6706	2,0003	2,3901	2,6603	3,2317	3,4602
70	0,0000	0,2543	0,5268	0,8468	1,2938	1,6669	1,9944	2,3808	2,6479	3,2108	3,4350
80	0,0000	0,2542	0,5265	0,8461	1,2922	1,6641	1,9901	2,3739	2,6387	3,1952	3,4164
90	0,0000	0,2541	0,5263	0,8456	1,2910	1,6620	1,9867	2,3685	2,6316	3,1832	3,4019
100	0,0000	0,2540	0,5261	0,8452	1,2901	1,6602	1,9840	2,3642	2,6259	3,1738	3,3905
200	0,0000	0,2537	0,5252	0,8434	1,2858	1,6525	1,9719	2,3451	2,6006	3,1315	3,3398
∞	0,0000	0,2533	0,5244	0,8416	1,2816	1,6449	1,9600	2,3263	2,5758	3,0903	3,2906

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Master of Financial Engineering - M1
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Statistics For Finance 1
Final Exam - Duration : 1 h45

No documents- You Can use a calculator

There is only one correct answer in Exercise 1.

Exercise 1. Multiple choice questions

1. In a simple linear regression of a variable x by another one y , if the coefficient of determination R^2 is 1, are the points $(x_i; y_i)$ aligned ?

☐ A No.
☒ B Yes.
☐ C Not necessarily.

2. Does the Least Squares line of a simple regression pass through the point $(\bar{x}; \bar{y})$. Where $\bar{x}$ and $\bar{y}$ are the respective means of the variables x and y of interest.

☒ A Always.
☐ B Sometimes.
☐ C Never.

3. The correlation between two variables x and y is negative. So,

☐ A If one variable gets bigger, the other variable tends to get bigger.
☒ B If one variable gets bigger, the other variable tends to get smaller.
☐ C None of the two responses is true.

4. In linear regression, the residual is

☒ A The difference between the observed value of the dependent variable and the predicted value of the dependent variable. That is, $|\hat{y}_i - y_i|$.
☐ B The difference between the observed value of the dependent variable and the explained variables $|x_i - y_i|$
☐ C The difference between the observed value of the dependent variable and its mean $|y_i - \hat{y}|$

Exercise 2. A machine is set up such that the average content of juice per bottle equals μ . Assume that the population standard deviation $\sigma = 5$ cl.

1. A sample of 100 bottles yields to an average content of 48cl. Calculate a 90% and a 95% confidence interval for the average content.

2. If the sample size is unknown; the average contents to be 0,5 at 95% IC:

$$48,98 = 0,5 + 1,96 \times \frac{S}{\sqrt{n}}$$

$$\Rightarrow 48,98 - 0,5 = 1,96 \times \frac{S}{\sqrt{n}}$$

$$\Rightarrow \sqrt{n} = \frac{1,96 \times 5}{(48,98 - 0,5)} = \frac{9,8}{48,48} = 0,202 \Rightarrow n = (0,202)^2$$

$$n = 0,0408$$

3. Test:

$$H_0: \mu = 50$$

$$H_1: \mu \neq 50$$

$$Z = \frac{48,5 - 50}{\frac{5}{10}} = - \frac{1,5}{0,5} = -3$$

$$P(-3 < Z) = P(3 > Z) = 1 - P(Z < 3) = 1 - 0,998 = 0,002 \times 2 = 0,004 < 0,05$$

$\Rightarrow 0,4\%$

We reject the hypothesis H_0

Exercise 3: $\hat{p} = 0,55$

$$p = 0,41$$

$$\alpha = 0,05$$

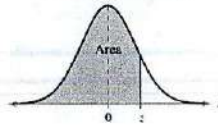
$$H_0: \hat{p} = 0,55$$

$$H_1: \hat{p} < 0,55 \text{ (or } \hat{p} \neq 0,55)$$

for n large!!

$$t = \frac{p - \hat{p}}{\sqrt{\hat{p}(1 - \hat{p})}} = \frac{0,41 - 0,55}{\sqrt{0,55(0,45)}} = - \frac{0,14}{0,12} = -1,66$$

Cumulative Distribution Function of the Standard Normal Distribution



Exemple : $P(Z \leq 1.96) = 0.97500$ se trouve en ligne 1.9 et colonne 0.06

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56750	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59484	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67365	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69498	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72241
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76731	0.77035	0.77337	0.77637	0.77935	0.78231	0.78524
0.8	0.78815	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82382	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84135	0.84375	0.84614	0.84850	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90148
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92786	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93575	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95544	0.95637	0.95728	0.95819	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97933	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99897	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976